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Nonparametric measurement of productivity and efficiency in education

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Abstract Nondiscretionary environmental inputs are critical in explaining relative efficiency differences and productivity changes in public sector applications. For example, the literature on education production shows that school districts perform better when student poverty is lower. In this paper, we extend the nonparametric approach to decompose the Malmquist Productivity Index suggested by Färe et al. (American Economic Rewiew 84:66–83, 1994) into efficiency, technological and environmental changes. The approach is applied to analyze educational production of Ohio school districts. Applying the extended approach in an analysis of the educational production of 604 school districts in Ohio, we find changes in environmental harshness are the primary drivers in productivity changes of underperforming school districts, while technical progress drives the performance of top performing school districts.

Keywords Data envelopment analysis · Nondiscretionary inputs · Productivity

1 Introduction

Beginning with the Coleman Report (1966), a substantial body of literature has examined the importance of socio-economic variables in the production of education. Hanushek (1979, 1986) discussed the importance of parental and student characteristics in determining desirable school outcomes. A useful framework for analyzing local public production is the Bradford et al. (1969) model where exogenous factors influence the transformation of government activities into desirable service outcomes. In education, for example, most students in an adverse environment will not be able to achieve the same outcomes as students in a more favorable learning environment, Connell (1994). This insight has important implications for the measurement of efficiency; see Ruggiero (1996).

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In this paper we investigate educational productivity and efficiency via nonparametric methods. The nonparametric literature on performance evaluation has an important starting point in the pioneering work of Farrell (1957), who developed a piecewise linear approximation to input isoquants and a decomposition of overall inefficiency into technical and allocative parts. His assumption of constant returns to scale (CRS) was later relaxed by Farrell and Fieldhouse (1962), who suggested a linear programming model. The extension to multiple outputs was suggested by Boles (1971) and popularized and named Data Envelopment Analysis (DEA) by Charnes et al. (1978). Färe and Lovell (1978) introduced the non-radial Russell measure that overcame excess slack inherent in the Farrell-based models.[1]

Literature on refining the DEA model includes Afriat (1972) and later Banker et al. (1984), who extended the approach to allow efficiency evaluation relative to a variable returns to scale frontier, and Banker and Morey (1986) and Ruggiero (1996), who allowed nondiscretionary inputs using a single-stage DEA model. Nonparametric methods have also been developed to measure productivity by analyzing performance in multiple time periods. Nishimizu and Page (1982) proposed the first decomposition of total factor productivity change, and Färe et al. (1992, 1994) developed the nonparametric estimation of the factors of productivity change by integrating Shephard's input distance function and the Malmquist Productivity Index (MPI) proposed by Cave et al. (1982). The index, a geometric mean of two distance functions, can be decomposed into a measure of change in efficiency and a measure of change in technology.

Extending this concept, several researchers developed further decompositions of productivity change. Ray and Desli (1997) proposed an alternative decomposition of the MPI in which technical change is measured relative to a variable returns-to-scale frontier. Sueyoshi and Aoki (2001) used a window analysis approach in which multiple time periods are aggregated into a "window" and technical change is measured via the MPI decomposition between periods. The size of the window is selected to reduce the effect of sampling on estimates of technical change. Lovell (2003) developed the decomposition of productivity change to more accurately capture scale effects, while also advocating a simultaneously oriented model.

Alternative methods have also been proposed for identifying the reference set in panel data. Tulkens and Vanden Eeckaut (1995) described three types of models to analyze panel data: contemporaneous, sequential and pooled. Contemporaneous models analyze each period individually as in the MPI originally proposed by Fare et al., but tend to attribute random sampling differences to technical change, and thus overestimate the effect of technical change. Sequential models include observations in prior periods in the reference set for measuring efficiency, which implies that technology can only progress and that technical regress is not possible. Pooled models aggregate data from all periods and analyze them in a single cross-section analysis which makes it impossible to measure technical change.

The MPI has been applied to investigate changes in productivity in industries such as: *healthcare* Rouse and Swales (2006); *pharmacies* Althin et al. (1996); *retailing* Vaz et al. (2010); *banking* Grifell-Tatjé and Lovell (1997); and *education* Ouellette and Vierstraete (2010). Each industry requires slightly different modeling approaches.

We consider public sector production that is characterized by nondiscretionary inputs. Building on Banker and Morey (1986) and Ruggiero (1996) in the education sector, we decompose overall productivity as a change in efficiency, technology and environmental

[1] Førsund and Sarafoglou (2005) provide an interesting and useful discussion of the history of DEA in the economics and operations research literature.

conditions. We then apply the decomposition to analyze the educational production of 604 school districts in Ohio, ranging from well-endowed to impoverished.

The remainder of this paper is organized as follows. In the next section, we introduce the efficiency measures based on distance functions for environments with and without nondiscretionary inputs. Section 3 presents the standard productivity decomposition and extends it to the public sector. In Sect. 4, we illustrate our decomposition with an application to Ohio school districts. Section 5 discusses our findings, conclusions, and future research directions.

2 Production technology and output-oriented efficiency measurement

2.1 Technology representation without nondiscretionary inputs

We first consider a production technology at time t where N decision-making units (DMUs) utilize a vector $X^t = (x_1^t, \dots, x_M^t)$ of M inputs to produce a vector $Y^t = (y_1^t, \dots, y_S^t)$ of S outputs.[2] Observed inputs and outputs for DMU$_n$ are $X_n^t = (x_{n1}^t, \dots, x_{nM}^t)$, and $Y_n^t = (y_{n1}^t, \dots, y_{nS}^t)$, respectively. We use the following convention throughout this paper: $Y_i^t \geq Y_j^t$ iff $y_{is}^t \geq y_{js}^t \forall s$ and $y_{is}^t > y_{js}^t$ for some $s = 1, \dots, S$. We represent production with the following empirical production possibility set (PPS) at time t:

$$T^t = \left\{ (Y^t, X^t) : \quad \sum_{n=1}^{N} \lambda_n y_{ns}^t \geq y_s^t, \quad s = 1, \dots, S; \quad \sum_{n=1}^{N} \lambda_n x_{nm}^t \leq x_m^t, \quad m = 1, \dots, M; \quad \lambda_n \geq 0, \quad n = 1, \dots, N \right\}. \tag{1}$$

We illustrate the empirical PPS in Fig. 1, where we assume one input x^t is used to produce one output y^t. The empirical PPS provides the boundary that allows us to measure technical efficiency. To do so, we introduce the following definition[3]:

Definition 1 $D^t(Y_j^t, X_j^t) = (\max\{\theta : (\theta Y_j^t, X_j^t) \in T^t\})^{-1}$ is the output-oriented measure of efficiency for $(X_j^t, Y_j^t) \in T^t$ assuming CRS.

From Fig. 1, we see $D^t(y_A^t, x_A^t) = a/b$. Note that a smaller value of $D^t(y_A^t, x_A^t)$ indicates more inefficiency. Using the same amount of input, observe that DMU B produces more output than DMU A.

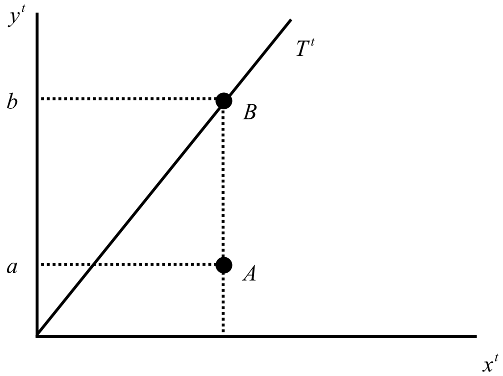
2.2 Technology representation with nondiscretionary inputs

We now extend the technology to allow nondiscretionary inputs. Suppose that each of the N DMUs described above faces an index z that captures the influence of non-discretionary in-

[2] We assume the inputs and outputs in the production function are stable over time. For an alternative concept see Chen and Johnson (2010).

[3] We present output-oriented efficiency using Shephard's distance function.

Fig. 1 Production technology



puts.[4] At time t, DMU$_n$ uses discretionary inputs X_n^t, non-discretionary input z_n^t and outputs Y_n^t, respectively. We represent production with the following empirical PPS with existing technology at time t:

$$T_z^t = \left\{ (Y^t, X^t, z^t): \quad \sum_{n=1}^{N} \lambda_n y_{ns}^t \geq y_s^t, \quad s = 1, \ldots, S; \right.$$
$$\sum_{n=1}^{N} \lambda_n x_{nm}^t \leq x_m^t, \quad m = 1, \ldots, M; \tag{2}$$
$$\lambda_n = 0 \quad \text{if } z_n^t > z^t, \; n = 1, \ldots, N;$$
$$\left. \lambda_n \geq 0, \quad n = 1, \ldots, N \right\}.$$

The technology in (2) is characterized by CRS with respect to the discretionary inputs and by a monotonic relationship between the nondiscretionary input and output,[5] *ceteris paribus*.

Banker and Morey (1986) showed that $D^t(Y_j^t, X_j^t)$ produces a biased measure because it is composed of both efficiency as well as the effect on output by the non-discretionary variable. The output-oriented measure of efficiency consistent with a technology characterized by non-discretionary inputs is:

Definition 2 $D^t(Y_j^t, X_j^t, z_j^t) = (\max\{\theta : (\theta Y_j^t, X_j^t, z_j^t) \in T_{z_j^t}^t\})^{-1}$ is the output-oriented measure of efficiency for $(X_j^t, Y_j^t, z_j^t) \in T_{z_j^t}^t$.

Figure 2 illustrates the measurement of efficiency with technology characterized by (2). We assume that $z_B^t > z_A^t$,indicating that DMU B has a more favorable environment. As Fig. 2 shows, DMU B is technically efficient while DMU A is technically inefficient. Follow-

[4] The selection of an overall index to reflect environmental influence is for expositional convenience only. In the case of multiple nondiscretionary inputs, Ruggiero (1998) and Estelle et al. (2010) provide alternative two-stage models to aggregate all nondiscretionary inputs in a single index.

[5] Banker and Morey (1986) impose convexity with respect to the non-discretionary variables, but (2) does not. For a more general model, see Johnson and Kuosmanen (2009).

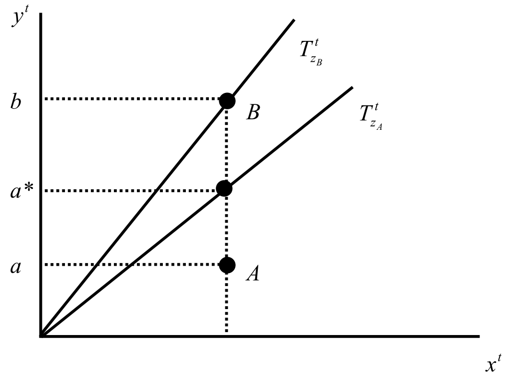


Fig. 2 Production technology with nondiscretionary inputs

ing Ruggiero (2000), we define the environmental harshness associated with the nondiscretionary input using the ratio of the distance functions above:

Definition 3 $E^t(Y^t_j, X^t_j, z^t_j) = D^t(Y^t_j, X^t_j)/D^t(Y^t_j, X^t_j, z^t_j) \leq 1$ is a measure of environmental harshness.

Returning to Fig. 2, we observe $D^t(y^t_A, x^t_A) = \frac{a}{b}$ and $D^t(y^t_A, x^t_A, z^t_A) = \frac{a}{a^*}$, resulting in $E^t(y^t_A, x^t_A, z^t_A) = \frac{aa^*}{ba} = \frac{a^*}{b}$. Whereas the distance function from a point to the frontier provides a measure of inefficiency, $E^t(Y^t_j, X^t_j, z^t_j)$ measures the distance between frontiers due to the environment; thus, a smaller value of $E^t(Y^t_j, X^t_j, z^t_j)$ indicates a worse environment. From the definition:

$$D^t(Y^t_j, X^t_j) = E^t(Y^t_j, X^t_j, z^t_j) * D^t(Y^t_j, X^t_j, z^t_j), \tag{3}$$

which shows that the distance to the overall frontier is a product of environmental harshness and efficiency; the distance from this best practice frontier increases if the production environment worsens and/or inefficiency increases.[6] We use this relationship to decompose productivity for public sector applications in the following section.

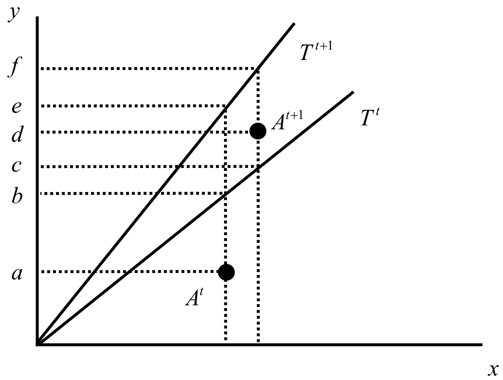
3 Measuring environmental productivity[7]

To measure productivity, we need to evaluate production plans in two different time periods. Therefore, we denote $D^{t+1}(Y^t_j, X^t_j) = (\max\{\theta : (\theta Y^t_j, X^t_j) \in T^{t+1}\})^{-1}$ as the distance function of DMU j in time t to the referent technology observed in time $t+1$ and $D^t(Y^{t+1}_j, X^{t+1}_j) = (\max\{\theta : (\theta Y^{t+1}_j, X^{t+1}_j) \in T^t\})^{-1}$ as the distance function of DMU j in time period $t+1$ to the referent technology observed in time period t. Färe et al. (1994)

[6]For ease of discussion, we refer to the frontier defined by technology (1) as the overall best practice frontier. We note that in a public sector application, this is equivalent to the frontier defined in (2) for units with the most favorable environment.

[7]While this paper focuses on the public sector and uses education as an example, we use the more general term, "environmental productivity", to recognize the existence of nondiscretionary factors of production in the private sector.

Fig. 3 Productivity growth



proposed an output-oriented MPI at period t relative to period $t+1$ as

$$MPI(Y_j^{t+1}, X_j^{t+1}, Y_j^t, X_j^t) = \left[\frac{D^t(Y_j^{t+1}, X_j^{t+1})}{D^t(Y_j^t, X_j^t)} \frac{D^{t+1}(Y_j^{t+1}, X_j^{t+1})}{D^{t+1}(Y_j^t, X_j^t)}\right]^{\frac{1}{2}}. \tag{4}$$

This measure can be decomposed as:

$$MPI(Y_j^{t+1}, X_j^{t+1}, Y_j^t, X_j^t) = \frac{D^{t+1}(Y_j^{t+1}, X_j^{t+1})}{D^t(Y_j^t, X_j^t)}\left[\frac{D^t(Y_j^{t+1}, X_j^{t+1})}{D^{t+1}(Y_j^{t+1}, X_j^{t+1})} \frac{D^t(Y_j^t, X_j^t)}{D^{t+1}(Y_j^t, X_j^t)}\right]^{\frac{1}{2}} \tag{5}$$

where the first term measures the change in efficiency from period t to period $t+1$ and the bracketed term measures the shift in technology.

Figure 3 illustrates the four distance function estimates we use to construct *MPI* for DMU A. In the absence of nondiscretionary inputs, we observe technology at time t and $t+1$. The distances to the period t frontier and the period $t+1$ frontiers are measured for both observed production levels. Consider first the change in efficiency. We estimate the efficiency of A in time $t+1$ as $D^{t+1}(Y_j^{t+1}, X_j^{t+1}) = d/f$. This is an improvement in efficiency from time t where efficiency was estimated as $D^t(Y_j^t, X_j^t) = a/b$. As a result, we observe $\frac{D^{t+1}(Y_j^{t+1}, X_j^{t+1})}{D^t(Y_j^t, X_j^t)} > 1$, indicating an improvement in efficiency for DMU A across time.

Next, we consider the measure of technical change as represented by the bracketed term. We calculate the distance of the data point (y_A^{t+1}, x_A^{t+1}) observed in time $t+1$ to the period t frontier as $D^t(y_A^{t+1}, x_A^{t+1}) = d/c$. Likewise, measuring the distance of the data point in time $t+1$ to the $t+1$ technology results in $D^{t+1}(y_A^{t+1}, x_A^{t+1}) = d/f$. Hence, our measure of productivity using $t+1$ data is $\frac{D^t(Y_j^{t+1}, X_j^{t+1})}{D^{t+1}(Y_j^{t+1}, X_j^{t+1})} = f/c > 1$, i.e., technical progress is observed. Using the data point in time t results in $\frac{D^t(Y_j^t, X_j^t)}{D^{t+1}(Y_j^t, X_j^t)} = e/b > 1$. To avoid arbitrarily choosing a particular time period in the calculations, the measures are averaged using the geometric mean.

The use of a CRS frontier by Färe et al. (1994) has been criticized in the literature by Ray and Desli (1997) and Lovell (2003). However, the proposed alternatives present other distinct problems, notably, Ray and Desli (1997) can lead to infeasible solutions and the mixed period terms that are ambiguous and difficult to interpret (Färe et al. 1997). Further as Kuosmanen and Sipiläinen (2009) address, the technical change component of the MPI characterizes the impact on total factor productivity (TFP) and not technical change. Productivity is only improved by shifts in CRS frontier, changes in variable returns to scale (VRS) efficiency cannot be distinguished from improvements in scale efficiency. Thus it is unclear and debatable whether the use of a VRS model can actually distinguish between technical change and scale efficiency components. For this reason we discuss our approach in terms of the CRS frontier.

The productivity measures (4) and (5) are derived for the case where production is not influenced by nondiscretionary factors. To develop an Environmental Malmquist Productivity Index (*EMPI*), we first note from (3) that

$$D^t(Y_j^t, X_j^t) = D^t(Y_j^t, X_j^t, z_j^t)E^t(Y_j^t, X_j^t, z_j^t). \tag{6}$$

Likewise,

$$\begin{aligned} &D^{t+1}(Y_j^{t+1}, X_j^{t+1}) = D^{t+1}(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1})E^{t+1}(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1}),\\ &D^{t+1}(Y_j^t, X_j^t) = D^{t+1}(Y_j^t, X_j^t, z_j^t)E^{t+1}(Y_j^t, X_j^t, z_j^t), \quad \text{and}\\ &D^t(Y_j^{t+1}, X_j^{t+1}) = D^t(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1})E^t(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1}). \end{aligned} \tag{7}$$

Substituting (6) and (7) into (5) and rearranging, we find:

$$\begin{aligned} EMPI(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1}, Y_j^t, X_j^t, z_j^t) &= \frac{D^{t+1}(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1})}{D^t(Y_j^t, X_j^t, z_j^t)} \frac{E^{t+1}(x_j^{t+1}, y_j^{t+1}, z_j^{t+1})}{E^t(x_j^t, y_j^t, z_j^t)}\\ &\times \left[\frac{D^t(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1})}{D^{t+1}(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1})} \frac{D^t(Y_j^t, X_j^t, z_j^t)}{D^{t+1}(Y_j^t, X_j^t, z_j^t)}\right]^{\frac{1}{2}}\\ &\times \left[\frac{E^t(x_j^{t+1}, y_j^{t+1}, z_j^{t+1})}{E^{t+1}(x_j^{t+1}, y_j^{t+1}, z_j^{t+1})} \frac{E^t(x_j^t, y_j^t, z_j^t)}{E^{t+1}(x_j^t, y_j^t, z_j^t)}\right]^{\frac{1}{2}}. \end{aligned} \tag{8}$$

Equation (8) shows that the *EMPI* can be decomposed into:

efficiency change $\left(\dfrac{D^{t+1}(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1})}{D^t(Y_j^t, X_j^t, z_j^t)}\right)$,

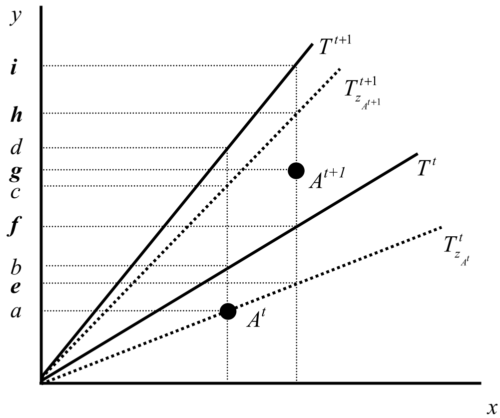
change in environmental harshness $\left(\dfrac{E^{t+1}(x_j^{t+1}, y_j^{t+1}, z_j^{t+1})}{E^t(x_j^t, y_j^t, z_j^t)}\right)$,

technical change $\left(\left[\dfrac{D^t(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1})}{D^{t+1}(Y_j^{t+1}, X_j^{t+1}, z_j^{t+1})} \dfrac{D^t(Y_j^t, X_j^t, z_j^t)}{D^{t+1}(Y_j^t, X_j^t, z_j^t)}\right]^{\frac{1}{2}}\right)$ and

environmental technical change $\left(\left[\dfrac{E^t(x_j^{t+1}, y_j^{t+1}, z_j^{t+1})}{E^{t+1}(x_j^{t+1}, y_j^{t+1}, z_j^{t+1})} \dfrac{E^t(x_j^t, y_j^t, z_j^t)}{E^{t+1}(x_j^t, y_j^t, z_j^t)}\right]^{\frac{1}{2}}\right)$.

The technical change terms in (8) use the geometric mean to calculate frontier shifts similar to the standard *MPI*. Alternative decompositions including scale proposed by Ray and Desli (1997) and Lovell (2003) can also be extended in an analogous manner to the Färe et al. (1994) *MPI*.

Fig. 4 Public sector productivity



The *EMPI* is illustrated in Fig. 4, where we observe production possibility A in times t and $t+1$. For convenience, the dashed lines represent the production frontier that is dependent on the non-discretionary inputs, the solid lines represent best practice frontiers under the most favorable environmental conditions, and bold letters indicate output levels in time $t+1$. Technical progress is shown because the production possibility set is larger in time period $t+1$. Note that production possibility A is technically efficient in time t but inefficient in $t+1$. The *EMPI* for A is given by:

$$EMPI = \frac{(g/h)}{1}\frac{(h/i)}{(a/b)}\left[\frac{h}{e}\frac{c}{a}\right]^{\frac{1}{2}}\left[\frac{(e/f)}{(h/i)}\frac{(a/b)}{(c/d)}\right]^{\frac{1}{2}}.$$

Efficiency change is given by $\frac{(g/h)}{1}$, where the denominator indicates that A is efficient in time t but is only $(g/h)100$ percent efficient in time $t+1$. Since the efficiency change is less than unity, we observe a decline in A's efficiency. The change in environmental harshness is measured by $\frac{(h/i)}{(a/b)} > 1$;[8] the numerator (denominator) measures the loss in output due to a relatively harsh environment in time $t+1(t)$. Hence, this ratio provides a measure of the change in the proximity of an efficient production frontier (accounting for the environmental conditions) to the best-practice frontier from one time period to the next. As illustrated in Fig. 4, the adverse effect of the nondiscretionary input for production possibility A is reduced from t to $t+1$ because h is relatively closer to i than a is to b. Importantly, the change in environmental harshness evaluates each production possibility in each time period without holding discretionary inputs fixed.

A measure of technical progress is given by $[\frac{h}{e}\frac{c}{a}]^{\frac{1}{2}} > 1$; the first ratio measures the productivity using the observation in time $t+1$. In this case, output expansion is possible from e to h due to technical progress, while controlling for the environment. Using the observation in time t to measure productivity, we also find technical progress with an expansion from a to c, holding the environment constant. Finally, environmental technical change is given by $[\frac{(e/f)}{(h/i)}\frac{(a/b)}{(c/d)}]^{\frac{1}{2}} < 1$, suggesting a reduction in the adverse effect of the environment. Using the data point in time $t+1$, we observe that environmental harshness is worse in time

[8]Note the change in harshness is greater than 1 in this example, but generally this does not have to be true. Similar statements hold for technical progress and environmental technical change described in the following paragraph.

t (e is farther from f) than in time $t+1$ (h is closer to i). Hence, $\frac{(e/f)}{(h/i)} < 1$ indicates that, technologically, the environmental effect on production is mitigated across time. Likewise, we observe the same tendency using the data point in time t as $\frac{(a/b)}{(c/d)} < 1$. For both technical change measures, we use the geometric mean.

4 Analysis of educational production

We empirically illustrate the productivity concepts defined above with an application to education. We consider State of Ohio kindergarten through twelfth grade school district performance for school years 2006 (t) and 2007 ($t+1$). In Ohio, numerous indicators of performance, including results on standardized tests, are assessed at various levels and school district performance is measured by an aggregate measure.

We select four classes of expenditures per pupil as inputs: administrative, instructional, building operation, and pupil support. Given that expenditures are also determined not only by input quantities but also by input prices, we deflate the expenditures by an index of first-year teacher salaries.[9] After controlling for input prices, the resulting deflated expenditures represent inputs. We also consider the percent of students in poverty as a nondiscretionary input.[10] For output, we use an index of student performance developed by the State of Ohio, which aggregates the measure of 30 statewide outcome goals including standardized tests in an overall measure of performance. This measure is highly correlated with individual outcomes. For an alternative model of education production see Thanassoulis and Portela (2002). The inputs are measured on a per student basis which would typically imply a CRS assumption. Since the output variable is an index of student performance, we assume that all schools should be able to achieve a similar level of performance in terms of the output index regardless of the number of students in each school. Our assumption is supported by the wide distribution of sizes of high schools identified as top performers via their test scores by *U.S. News and World Report* 2009. Descriptive statistics are reported in Table 1.

The State of Ohio classifies school districts based on the number of standards met. The designations from school year 2007–2008 include Academic Watch, Continuous Improvement, Effective, Excellent, and Excellent with Distinction.[11] Means and standard deviations of the districts for each designation are reported in Table 1. The performance scores (output) decrease in the designation of the school. Note that the Academic Watch group comprises only 1.5% of the observed schools and thus has significantly larger standard deviations than the other designated groups. We observe that on average all groups except Continuous Improvement experience a decrease in the average resources available. We find that, due to the multiple input nature of the data and nondiscretionary poverty input, it is difficult to draw conclusions about the appropriateness of the designated groups, and if lack of resources or an abundance of poverty drives the lower performance scores.

We use DEA and the *EMPI* decomposition described above to investigate the sources of changes in productivity. The results are reported in Table 2. We find that the average efficiency for all groups is similar in 2006 and 2007, ranging between 0.80 and 0.85. With the exception of the districts designated as Excellent with Distinction and Academic Watch,

[9] We use beginning teacher salaries from school year 2004–2005.

[10] For the DEA models, we use (100-poverty rate) as the nondiscretionary factor, to be consistent with our theoretical model. Hence, an increase in the non-poverty rate leads to higher outcomes, *ceteris paribus*.

[11] No school districts were designated Academic Emergency, the lowest designation. Our analysis was developed independently of this classification.

Table 1 Descriptive statistics

All districts ($N = 604$)	2006–2007		2007–2008	
	Mean	Std. Dev.	Mean	Std. Dev.
Performance Score	95.64	6.05	95.82	6.31
Admin Expenditure per Pupil	1,196.10	264.23	1,122.97	316.49
Building Operation Exp. per Pupil	1,906.37	429.91	1,786.22	441.63
Instructional Exp. per Pupil	5,400.14	868.57	5,018.06	722.53
Pupil Support Exp. per Pupil	972.46	312.48	897.02	256.89
Poverty Rate	3.05	3.49	3.04	3.35
Academic Watch ($N = 9$)				
Performance Score	76.10	3.53	74.91	3.12
Admin. Exp. per Pupil	1,651.41	395.07	1,473.59	342.41
Building Operation Exp. per Pupil	2,662.63	736.46	2,393.78	700.54
Instructional Exp. per Pupil	7,047.09	669.01	6,227.22	734.53
Pupil Support Exp. per Pupil	1,236.15	274.85	1,105.39	320.11
Poverty Rate	15.62	6.07	14.36	5.39
Continuous Improvement ($N = 82$)				
Performance Score	87.86	4.34	87.31	3.98
Admin Exp. per Pupil	1,265.88	304.98	1,308.34	425.07
Building Operation Exp. per Pupil	2,009.53	409.61	2,065.74	626.64
Instructional Exp. per Pupil	5,330.47	828.94	5,736.66	649.67
Pupil Support Exp. per Pupil	939.66	306.06	1,014.06	216.90
Poverty Rate	6.78	4.19	6.80	4.08
Effective ($N = 287$)				
Performance Score	94.97	3.32	94.94	3.05
Admin Exp. per Pupil	1,179.37	223.34	1,119.02	235.29
Building Operation Exp. per Pupil	1,854.47	364.67	1,775.21	391.89
Instructional Exp. per Pupil	5,205.97	694.82	4,955.10	701.20
Pupil Support Exp. per Pupil	924.02	265.63	871.96	229.41
Poverty Rate	2.71	2.56	2.73	2.48
Excellent ($N = 152$)				
Performance Score	98.66	4.30	99.48	4.27
Admin Exp. per Pupil	1,179.31	290.42	1,108.52	384.82
Building Operation Exp. per Pupil	1,875.75	505.99	1,731.39	391.26
Instructional Exp. per Pupil	5,343.05	945.19	4,958.56	697.61
Pupil Support Exp. per Pupil	976.37	385.33	901.38	326.93
Poverty Rate	2.02	1.98	2.01	1.89
Excellent with Distinction ($N = 152$)				
Performance Score	103.01	2.79	103.70	2.69
Admin Exp. per Pupil	1,115.69	196.39	966.99	166.17
Building Operation Exp. per Pupil	1,901.94	344.75	1,620.20	258.91
Instructional Exp. per Pupil	5,697.29	987.27	4,891.15	722.96
Pupil Support Exp. per Pupil	1,074.10	283.79	912.65	210.41
Poverty Rate	0.83	0.79	0.85	0.75

All calculations by authors

Table 2 Average efficiency and productivity results by classification

Designation	Efficiency		*EMPI*	Decomposition			
				Change in Env.		Technical change	
	t	$t+1$		Eff.	Harshness	Env.	Standard
Academic Watch	0.80	0.80	1.12	1.02	0.96	1.12	1.05
Continuous Improvement	0.81	0.82	1.07	1.02	0.91	1.11	1.05
Effective	0.84	0.80	1.06	0.96	0.96	1.04	1.11
Excellent	0.85	0.83	1.09	0.98	0.97	1.02	1.12
Excellent with Distinction	0.83	0.85	1.17	1.03	0.98	1.04	1.15
Grand	0.83	0.82	1.09	0.98	0.96	1.04	1.11

Output-oriented models are used. The *EMPI* and its decomposition follow (8). All calculations by authors

we observe similar average increases in the *EMPI* measure. Interestingly, the Excellent with Distinction districts are considered among the best performing and the Academic Watch districts are among the worst performing in the state. The Excellent with Distinction districts show a relatively large average increase in the *EMPI* of 17 percent and the Academic Watch districts show an average increase of 12 percent.

The decomposition of the *EMPI*, however, reveals different sources of the increased productivity. On average, we find a modest increase for the two groups of about 2 to 3 percent in efficiency and a slight decrease in the proximity to the best practice frontier. Hence, on average, these districts are able to compensate for a slightly more adverse environment with slight improvements in efficiency. The main difference between the groups is the role of technical change and environmental technical change. The Excellent with Distinctions districts show a 15 percent increase in output due to technical change. Likewise, they benefit from a relatively low worsening of environmental technical change. The Academic Watch districts, on the other hand, show the lowest improvement in technical progress, but a larger adverse effect of environmental technical change, on average.

Table 2 also reports other interesting results. For example, only the Effective and Excellent districts experience a decrease in average efficiency. Both the Academic Watch and Continuous Improvement districts realize technical progress, although at a smaller rate than the other districts. Of note, these two districts also show higher increases in environmental technical change, indicating that their operating environments grow relatively harsher.

The results of our approach indicate that the increase in environmental harshness due to the poverty rate largely explain the growth in public school productivity. These results support Hanushek (1986) and others who argue that environmental factors impact drive school performance more significantly than resource allocation. While we find there are technological advances in education, not all districts are able to realize these gains because of their environments. Our findings indicate that the average district realizes improved productivity despite a slight decline in technical efficiency. However, the decomposition approach also reveals that the average district realizes an increase in environmental harshness.

Table 3 provides illustrative results for a four representative districts: Cleveland, Toledo, Steubenville, and Oakwood. Other than suburban Oakwood, the other districts encounter high levels of poverty. Toledo, and especially Cleveland, are large city districts that face many of the problems found in the inner city. Of the three high-poverty districts, only Cleveland is classified by Ohio as an Academic Watch district (meeting only 2 of the 30 established standards) with among the worst outcome levels. Considering enrollment, Cleveland

Table 3 Comparison of four districts

Variable	School district			
	Cleveland	Toledo	Steubenville	Oakwood
	Academic watch	Continuous improvement	Effective	Excellent with Distinction
2007–2008 Classification				
Poverty Rate				
2006–2007	19.69	17.77	18.85	0.05
2007–2008	17.36	17.87	19.19	0.05
Outcome				
2006–2007	76.2	79.1	94.8	108.1
2007–2008	72.1	80.1	97.1	107.8
Results				
EMPI	1.19	1.29	1.00	1.33
Δ Efficiency	0.85	1.24	0.98	1.12
Δ Env. Harshness	1.19	0.93	0.88	1.00
Technical Change	1.06	1.01	0.96	1.18
Env. Technical Change	1.11	1.10	1.17	1.00

All calculations by authors

has over 50,000 students and is second to Columbus. In contrast, Toledo has 27,200 students with a similar poverty rate and Steubenville has about 2,200 students. Yet even with high levels of poverty, Steubenville still produces high outcomes (meeting 24 of the 30 standards). As the districts improve within the classification, we observe higher outcomes in both years, with Oakwood meeting all standards and achieving aggregate outcomes in the 99th percentile. With the exception of Steubenville, the other three districts have a high value of EMPI: Toledo (1.29), Oakwood (1.33) at about the 95th percent rank, and Cleveland (1.19). Decomposition provides insight into these values. Toledo and Oakwood both see improvements in efficiency, Steubenville becomes slightly less efficient, and Cleveland relatively more efficient. Cleveland's decrease in the poverty level leads to an improved operating environment in $t + 1$; however, possible gains are not realized due to increased inefficiency. Only Steubenville experiences technical regress, but the increase in poverty causes an increase in environmental harshness and environmental technical regress. Oakwood, on the other hand, maintains a very favorable environment with low poverty rates, technical progress, and an improvement in efficiency.

5 Conclusions

In this paper we extended the nonparametric measure of productivity to the public sector where nondiscretionary factors are important contributors to output. We introduced an Environmental Malmquist Productivity Index useful for public sector (and private sector) applications and decomposed it into efficiency change, environmental harshness change, and technical change. Technical change was further decomposed into changes in the effect that nondiscretionary variables have on production and technical change that occurs independent of environment. The new model was applied to analyze the educational production of 605

Ohio school districts in 2006 and 2007. Our results showed that the decomposition of productivity to account for environment provides information useful in analyzing performance.

Based on our findings, we suggest that future research should be undertaken to identify if these results persist across longer panel data sets and are applicable to other states. The index can easily and inexpensively be used to advise policy-makers of the true drivers of variations in performance within the public sector.

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