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Measuring Technical Efficiency in Sports

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Abstract

Standard economic production theory is the basis for measuring technical efficiency in sports. Using programming or regression models, efficiency is defined as the distance of a given team observation from the technology. In this article, the authors show that the standard measures of efficiency using deterministic models are biased downward due to serial correlation with respect to the efficiency measure. In particular, if the number of observed wins for a given team is affected by the team's inefficiency, it is necessarily true that another team is able to produce outside of the technology. As a result, the observed frontier is not feasible if all inefficiency is eliminated. In this article, the authors propose a correction to this problem and apply new models to estimate efficiency in professional football.

Keywords

technical efficiency, DEA, regression, serial correlation

Introduction

Measurement of managerial performance/efficiency in professional sports has an extensive list of empirical analyses in the sports economics literature. This type of research exists for Major League Baseball (e.g., Horowitz, 1994a, 1994b; Porter & Scully, 1982; Ruggiero, Hadley, & Gustafson, 1996), the National Football

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League (e.g., Hadley, Poitras, Ruggiero, & Knowles, 2000), the National Basketball League (Zak, Huang, & Siegfried, 1979), and Major League Soccer (Haas, 2003). Many of these studies model team performance (either wins or winning percentage) as a function of some measure of player inputs.

Hadley et al. (2000) model team wins in the National Football League as a function of offensive and defensive performance statistics (e.g., average yards per rush, percentage of passes intercepted, etc.) using Data Envelopment Analysis (DEA). Haas (2003) uses players' and coach's wages as the inputs and team points earned, average attendance, and total revenues as the output measures in a DEA model estimating efficiency in Major League Soccer. Einolf (2004) uses players' salaries as inputs and team wins, team batting average, and team earned run average as outputs in a production model of Major League Baseball. Additionally, Einolf (2004) models a production function for the National Football League using players' salaries as inputs and team wins, team offensive yards gained per attempt, and defensive yards given up per attempt as outputs.

The methods developed to measure managerial performance/efficiency have largely been developed in the field of production economics. However, the field is divided between two competing paradigms: the parametric approach that is based on the tools and concepts from the regression analysis, and the nonparametric approach that builds upon axiomatic properties and mathematical programming techniques. While both approaches stem from the same origins and share the same main objectives, the user communities and the empirical results have traditionally been different (see e.g., Forsund & Sarafoglou, 2002).

The roots of the nonparametric approach using mathematical programming are in the activity analysis pioneered by Koopmans (1951). From this line of research came the seminal paper by Farrell (1957), which established the standard radial input and output efficiency measures and decomposed the overall efficiency into components of technical and allocative efficiency. Farrell also proposed to estimate the production frontier by the most pessimistic piecewise linear envelopment of the data, calculated through solving a system of linear equations. Farrell's work is recognized as the point of origin for both the parametric and the nonparametric approaches (Forsund & Sarafoglou, 2002).

The parametric methods have roots in the Corrected Ordinary Least Squares (COLS) method first suggested by Winsten (1957) in comments to Farrell (1957); see also Richmond (1974). The method proposed to capture the mean behavior of the data through Ordinary Least Squares (OLS) and then shift the frontier up by the size of the largest residual in order to envelop the data set. This method along with the work of Aigner and Chu (1968) provided the basis for the subsequent development of the stochastic frontier approach (SFA) by Aigner, Lovell, and Schmidt (1977) and Meeusen and Vandenbroeck (1977).

Farrell's measure was extended in the nonparametric literature by Boles (1966, 1971), who implemented the method in linear programming techniques. Afriat (1972) developed the free disposal hull estimator and the variable returns to scale

(VRS) assumption to the nonparametric frontier estimators. The nonparametric approach was championed by Charnes, Cooper, and Rhodes (1978), who coined the catchy name data envelopment analysis (DEA). The influential work by Charnes et al. instituted DEA as the dominating approach in the field of operations research and management sciences. The axiomatic foundation of DEA also appeals to many theoretically minded economists, whereas the econometrians traditionally favor the parametric regression based techniques.

Both the parametric and the nonparametric methods have been used widely in the sports literature to measure efficiency of teams (see e.g., Dawson, Dobson, & Gerrard, 2000; Debrock, Hendricks, & Koenker, 2004 for parametric analyses and Hadley et al., 2000 and Einolf, 2004 for nonparametric analyses). Unfortunately, both types of models fail to account for the fact that one team's wins are affected by the efficiency (or inefficiency) of the teams they play against. Thus, serial correlation exists in the teams' inefficiency terms, which causes measurement of managerial performance to be biased in one direction or the other depending on their opponents' levels of efficiency. This bias has previously been discussed under the term zero-sum in the context of Olympic medals by Lins, Gomes, Soares de Mello, and Soares de Mello (2003). We extend this work in several directions by explaining which zero-sum models are appropriate in the sports league context where wins are the measure of performance; we describe how parametric estimators such as COLS and SFA can be adjusted; we discuss the influence of scheduling and propose methods to address incomplete round robin scheduling; methods to extend to the multiple output setting where not all outputs have serial correction are also discussed.

Technical Efficiency Measurement

We represent production in sports with a production function assuming that teams use a vector $X \equiv (x_1, ..., x_m)$ of m inputs to produce one output wins (W) according to the production function:[1]

$$W = f(X). \tag{1}$$

Standard assumptions regarding production are assumed, Färe and Primont (1995). Team j data are represented by W_j and $X_j = (x_{1j}, ..., x_{mj})$ for $j = 1, ..., n$. In the efficiency literature, Equation 1 represents frontier production; inefficiency is measured as deviations from this frontier. Standard production analyses extend Equation 1 by recognizing deviations from the frontier:

$$W = f(X) + \varepsilon, \tag{2}$$

where we assume for expositional convenience an additive inefficiency term ε. In the following subsections, we present alternative techniques for the measurement of technical efficiency.

DEA

DEA is a linear programming-based model that estimates efficiency ε with minimal assumptions on the technology. In particular, it is assumed that $\varepsilon \leq 0$ and that the production technology can be represented by convex sets. For a production function f estimated under the maintained assumptions of monotonicity and concavity (i.e., the DEA production function), the VRS DEA estimator of f can be formally defined as (Afriat, 1972; Banker, Charnes, & Cooper, 1984)

$$f^{\text{DEA}}(\mathbf{x}) = \max_{\lambda \in \Re_{+}^{n}} \left\{ y \,\middle|\, y = \sum_{j=1}^{n} \lambda_j y_j; \mathbf{x} \geq \sum_{j=1}^{n} \lambda_j \mathbf{x}_j; \sum_{j=1}^{n} \lambda_j = 1 \right\}. \quad (3)$$

The linear programming model to estimate the output-oriented efficiency TE_j of team j for $j = 1, ..., n$ is given by:

$$\begin{aligned} TE_j^{-1} &= \text{Max } \theta_j \; s.t. \\ &\sum_{k=1}^{n} \lambda_k W_k \geq \theta_j W_j \\ &\sum_{k=1}^{n} \lambda_k x_{ik} \leq x_{ij} \quad \forall i = 1, \ldots, m \\ &\sum_{k=1}^{n} \lambda_k = 1 \\ &\lambda_k \geq 0 \quad \forall k = 1, \ldots, n. \end{aligned} \quad (4)$$

The last two constraints insure minimum extrapolation of the data. Solving this linear program for each team leads to our measure of technical efficiency for all teams. The assumptions of convexity, monotonicity, free disposability, and minimum extrapolation guarantee a consistent estimator of efficiency (Banker, 1993).

An alternative DEA model is the additive model that projects a unit to the frontier based on slack and not as a radial expansion in outputs. Maintaining the assumption that the only desirable output is W, the additive output-oriented model under the assumption of VRS is given by:

$$\begin{aligned} S_j^{*} &= \text{Max } S_j \; s.t. \\ &\sum_{k=1}^{n} \lambda_k W_k - S_j = W_j \\ &\sum_{k=1}^{n} \lambda_k x_{ik} \leq x_{ij} \quad \forall i = 1, \ldots, m \\ &\sum_{k=1}^{n} \lambda_k = 1 \\ &S_j \geq 0 \\ &\lambda_k \geq 0 \quad \forall k = 1, \ldots, n. \end{aligned} \quad (5)$$

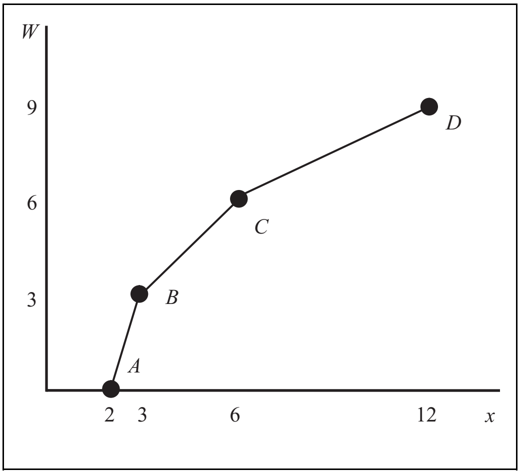

Figure 1. Efficient production (DEA).

The difference between Equations 5 and 4 is the specification of the objective function. In Equation 5, the maximum slack S_j^* between the observed production possibility is measured as opposed to a distance function. Hence, S_j^* represents the number of additional wins that team j could have achieved, given the same resource level. It is easy to show that $S_j^* = (TE_j^{-1} - 1)W_j$ in the single output case.

We illustrate the generation of the DEA frontier assuming one input x and output W. We further assume that four teams A–D compete in the league and play each other three times. For simplicity, we rule out tied games. Input and output data are shown in the following table:

Team	x	W
A	2	0
B	3	3
C	6	6
D	12	9

The data are shown in Figure 1. Team A uses the lowest amount of x and therefore wins 0 games. The assumption of monotonicity holds; as x increases, W increases. For this illustrative example, we assume that the team with the highest input level wins all nine games. In Figure 1, we represent the frontier using a piecewise linear frontier; similar implications hold if we represent the frontier with a smooth function. The resulting DEA frontier reveals the minimum input usage necessary to achieve a given number of wins.

We now consider the effect that inefficiency has on the estimated production frontier. In most efficiency applications, the inefficiency of the decision-making unit

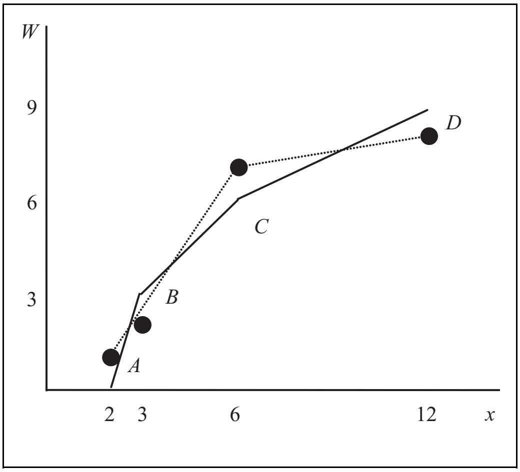

Figure 2. Observed production with inefficiency.

is independent of the other decision-making units' production plans. However, this does not hold in sports because the number of total wins equals the number of games played. If inefficiency causes a team to lose a game that it should have won, the opponent gets an additional win. We consider this in Figure 2, where we replicate Figure 1 but add inefficiency. In particular, we assume that teams *B* and *D* are inefficient and lose a game that should have been won. As a consequence, teams *A* and *C* gain a win. Importantly, this gain in wins results not from any change in input level but rather from inefficiency on the part of their opponents.

The effect that a team's inefficiency has on the estimated frontier is revealed; teams *A* and *C* have been displaced to an infeasible point beyond the frontier. With the inefficiency, *A* and *C* are able to produce more wins than possible, given their input usage only because of the inefficiency of *B* and *D*. The new frontier consists of line segments *AC* and *CD*. Team *B* can apparently increase the number of wins holding its input level constant relative to a convex combination placing *B* on a convex combination of *A* and *C* (output orientation) or it could reduce its input level holding its wins constant to identify a second benchmark also on the hyperplane defined by the convex combinations of *A* and *C*.

We will first consider the output orientation. If team *B* were to increase the number of wins while holding its input level constant, it is necessarily true that either *A, C*, or *D* must lose a game. This loss can happen if *A, C*, or *D* becomes (more) inefficient, uses less input *x* or in the case of *A* and *C*, loses a win that was gained from *B* or *D*. Holding input usage constant, we see that this can happen only with a change in efficiency of the other teams. Otherwise, if *B* is projected back to the correct frontier ceteris paribus, the total number of wins is greater than the number of games.

For the input orientation, team B would be projected to benchmark a convex combination of A and C. However, this projection is an infeasible point (relative to the true efficiency frontier) that is possible only because A and C have taken advantage of the inefficiency of other teams. After projection, we would have what appears to be efficient production. But this cannot be achieved; A's observed production resulted from inefficiency of another team. Holding all teams efficient, with $x = 2$, the maximum amount of wins is $W = 0$.

COLS

COLS is a deterministic approach that estimates Equation 2 using regression after assuming a parametric functional form. COLS is a two-stage procedure: in the first stage, the frontier is estimated by OLS regression; in the second stage, the frontier is shifted upward such that the resulting COLS frontier envelops all data. For expositional convenience, we assume a Cobb-Douglas form:

$$\text{Ln } W = \alpha + \beta_1 \text{Ln } x_1 + \ldots + \beta_m \text{Ln } x_m - \mu, \tag{6}$$

and estimate using OLS. The resulting error term μ is two-sided. Greene (1980) proves that the slope parameters are estimated consistently while the intercept term is biased downward. However, a consistent estimate of α can be obtained by correcting the intercept and adding the largest positive residual. As a result, all adjusted residuals are non-positive, leading to an efficiency measure of:

$$TE_j = \exp(-\hat{\mu}_j), \textit{where } \hat{\mu}_j = \mu_j - \max_i(\mu_i). \tag{7}$$

We note that the contradiction discussed about DEA also applies to the COLS measure; shifting the frontier up to the point that has the largest residual will lead to a frontier that has infeasible points. This is illustrated in Figure 3, where the true production frontier is superimposed under the COLS frontier. In this case, COLS identifies A and C as efficient and inefficiency is measured relative to the deviation in the W dimension. Of course, given the parametric specification, moving from input orientation to output orientation is trivial. However, we note that the two teams that comprise the COLS frontier are actually infeasible if the inefficiency from other teams is removed.

SFA

An alternative regression-based approach, the stochastic frontier approach (SFA), assumes that ε from Equation 2 is composed of measurement error and inefficiency. Shown below in Equation 8 for the Cobb-Douglas form:

$$\text{Ln } W = \alpha + \beta_1 \text{Ln } x_1 + \ldots + \beta_m \text{Ln } x_m - \mu + v, \tag{8}$$

where μ is assumed to be a one-sided inefficiency term and v is a two-sided noise component. With a priori assumptions on the distributions, maximum likelihood

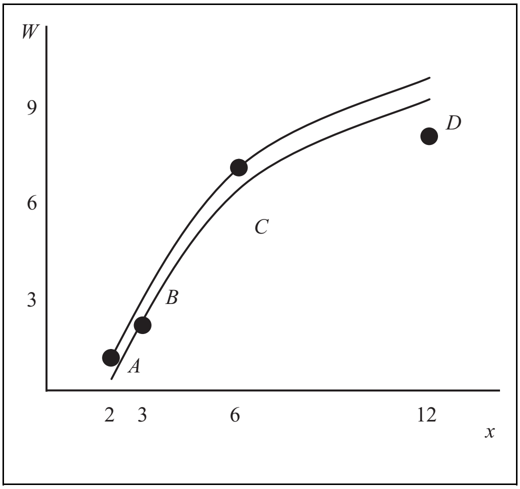

Figure 3. Estimating production with COLS.

estimation is employed and a measure of inefficiency based on conditional expectations is used. Ondrich and Ruggiero (2001) show that the cross-sectional estimator is not consistent. However, like COLS, the SFA model assumes a homoscedastic error structure and focuses on adjusting the intercept term to correct for the presence of inefficiency. The intercept can be corrected using the following equation,

$$\hat{\alpha}' = \hat{\alpha} + E(\mu) = \hat{\alpha} + \frac{\sqrt{2}}{\sqrt{\pi}}\sigma_{\mu}, \tag{9}$$

where σ_{μ} can be estimated through the method of moments described in Aigner et al. (1977).

Output-Oriented Adjustment for Serial Correlation

The problem with current efficiency models is that infeasible points arising from serial correlation with respect to efficiency determine the efficient frontier. An additional assumption that the total number of frontier wins must equal the total number of games played. In this section, we provide models that overcome this weakness.

DEA Under Serial Correlation

First, we develop a procedure to correct the DEA efficiency scores. In the solution of Equation 5, we obtain $S_j^* \; \forall j = 1, \ldots, n$. In the case of inefficiency affecting the wins and losses, the resulting distribution should be mean zero. However, because $S_j^* \geq 0$, the resulting distribution is one-sided. A solution to this problem is to adjust the resulting biased efficiency scores obtained in Equation 5 by subtracting a fixed constant:

$$C = \frac{1}{n}\sum_{j=1}^{n} S_j^*. \tag{10}$$

The resulting index $\tilde{S}_j^* = S_j^* - C$ becomes an unbiased estimate of the slack; it is straightforward to show that the mean of $\tilde{S}_j^*$ is zero. Efficiency is then calculated as:

$$TE_j = \begin{cases} 1 & \text{if } \tilde{S}_j^* \leq 0, \\ \dfrac{W_j}{W_j + \tilde{S}_j^*} & \text{otherwise.} \end{cases} \tag{11}$$

We note that if $\tilde{S}_j^* < 0$ model (5) incorrectly classified a unit as inefficient due to the upward bias of the frontier. In particular, an efficient team could be identified as inefficient if it is projected to a benchmark that would be infeasible absent other teams' inefficiency. After the correction, a correct classification of efficient can be achieved.

Lins et al. (2003) identifies two strategies for shifting the frontier: equal output reduction and proportional output reduction. The method we are advocating is a type of equal output reduction method. The authors identify two drawbacks of the equal reduction method: their formulation of the method is a nonlinear program and the output level for the adjusted frontier could have negative output levels. The two-stage approach we have suggested only requires a linear program in the first stage and arithmetic in the second stage, addressing the first concern.

The issues of negative outputs, however, is possible. Note the frontier for a given input level is simply a point estimate. In the parametric cases, a standard confidence interval would typically include zero or a positive value. While DEA has often been referred to as a deterministic method, recently Simar and Wilson (2008), have described DEA statistical properties based on a particular observed sample. Thus while it is less common, similar confidence intervals could be developed for the DEA estimator. Therefore, we do observe the same drawbacks that Lins et al. (2003) identify as their motivation for a proportional adjustment strategy. Further, following the work of Färe and Zelenyuk (2003), we prefer the additive DEA model or regression models in which the inefficiency term is additive because it facilitates aggregation of multiple teams to a division and divisions to a particular league (i.e., AFC). The use of multiplicative or proportional adjustments makes the comparisons of aggregates inconsistent with individual team analysis. See Färe and Zelenyuk (2003) for more details.

Often it is possible for a team to pursue multiple objectives and thus have more than one output used in determining their efficiency. While parametric methods require that weights be given a priori to aggregate to a single utility function, a benefit of DEA is the ability estimate a multi-input and multi-output production function without the specification of weights. To this point we have assumed the single output

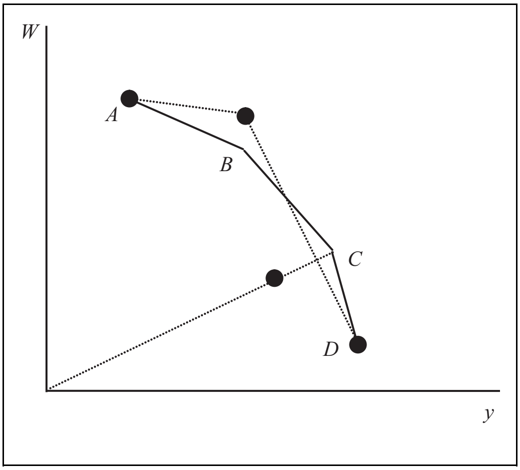

Figure 4. Production with multiple outputs.

for a sports team is wins, but we could imagine that profits may also be an important output. In this situation, the output profit would not have the same serial correlation in a cross-sectional model as the output wins. Thus, we present a general model in which a team can have multiple outputs, some of which are serially correlated while others are not.

Multiple Outputs and DEA

For the multiple output case, we assume a production process where a team uses a vector $X \equiv (x_1, \ldots, x_m)$ of m inputs to produce a vector $Y \equiv (y_1, \ldots, y_{s-1}, W)$ of s outputs where $s - 1$ of the outputs are not serially correlated with respect to other teams' inefficiency. We maintain our previous assumption regarding inefficiency and the sth output (W). Team j data are represented by $Y_j \equiv (y_{1j}, \ldots, y_{(s-1)j}, W_j)$ and $X_j = (x_{1j}, \ldots, x_{mj})$ for $j = 1, \ldots, n$.

The effect that serial correlation has is illustrated in Figure 4 assuming two outputs are produced. Four teams *A–D* are observed producing various levels of the two outputs using the same aggregate input level. The solid piecewise linear frontier is derived assuming no inefficiency. Now we assume that team C is Farrell inefficient, producing less of both outputs. As a result, C lost a game that should have been won; team B gained the extra win. The effect on the frontier is similar to the input–output case; in this case, the distance between the observed and frontier wins for B is the same as the vertical distance between the observed and frontier production for team C. The effect is infeasible points in the observed piecewise linear frontier; convex combinations of A and B (excluding A), for example, are not feasible for the given input level.

In order to estimate the efficiency of teams in the multiple output case, we appeal to the Banker and Morey (1986) model. This model was developed to allow efficiency estimation when some production variables are fixed and beyond the control of the decision-making unit. For our purpose, the model is extended to measure efficiency in the win dimension. The linear programming model to evaluate the efficiency of team j is given by:

$$\begin{aligned} S_j^* = \text{Max } & S_j \; s.t. \\ & \sum_{k=1}^{n} \lambda_k W_k - S_j = W_j \\ & \sum_{k=1}^{n} \lambda_k y_{lk} \geq y_{lj} \quad \forall l = 1, \ldots, s-1 \\ & \sum_{k=1}^{n} \lambda_k x_{ik} \leq x_{ij} \quad \forall i = 1, \ldots, m \\ & \sum_{k=1}^{n} \lambda_k = 1 \\ & S_j \geq 0 \\ & \lambda_k \geq 0 \quad \forall k = 1, \ldots, n. \end{aligned} \tag{12}$$

Model (12) differs from the single output model (5) with the inclusion of additional convexity constraints on the $s - 1$ outputs that are correlated with other team's inefficiency. Using (10) and (11), we can then measure efficiency using the same correction that was used in the single output case.

COLS Under Serial Correlation

While the regression-based approaches can only be used in the single output case, the correction to COLS is more straightforward and can also be applied to SFA. In the efficiency literature, the residual from the OLS regression is corrected via Equation 7 in COLS. In the case of the SFA, moments of the OLS residuals are used to correct the bias of the OLS residual. See Coelli (1995) for details. However, the OLS residuals are correct in the presence of serially correlated inefficiency. As a result, Equation 5 is estimated via OLS; the resulting efficiency estimate is obtained similar to the DEA case (7):

$$TE_j = \begin{cases} 1 & \text{if } \mu_j \geq 0, \\ TE_j = \exp(-\mu_j) & \text{otherwise.} \end{cases} \tag{13}$$

In the case of these efficiency models, positive residuals indicate efficient production above the frontier while negative residuals indicate technical inefficiency. Using either the COLS or SFA models without the correction would lead to biased efficiency estimates.

Minimizing the Sum of Absolute Errors (SAE)

The concept of COLS can be applied to other estimators in addition to OLS (Kuosmanen & Johnson, 2010). For example, the SAE[2] minimizes the L1-norm rather than the L2-norm in standard OLS. SAE was first suggested by Boscovich in 1757 (Koenker & Bassett, 1985) and later studied by Laplace before least squares was developed by Legendre (1805).[3] The SAE is formulated as

$$\underset{f_i(x_{j_i})\in\Re^n,\mu\in\Re^n_+}{\arg\min} \sum_{j_i} \left[y_{j_i} - f_i\left(\mathbf{x}_{j_i}\right)\right]. \tag{14}$$

The SAE estimator can be used as a frontier estimator when output levels across observations are serially correlated. This estimator has several nice properties.

It often occurs in professional sports that teams in a league or division do not play every other team in that league or division. In this case, the analyst may be concerned that the serial correlation results are not as severe.[4] It is also possible that there are differences across divisions (such as rules, fields, weather conditions, etc.) that may affect game results. The analyst may be interested in estimating the league production function as the average across the divisions, given these division-level differences. The SAE estimator has the desirable property that the production function estimated as the average across division production functions is exactly equal to the production function estimated using all teams in the league. This insight is formalized in the following theorem:

Theorem 1: Using the SAE estimator, the league production function estimated as the average of division production functions is equal to the league production function estimated using all teams in the league.
Proof: see appendix.

While this result holds for the SAE estimator with an equal reduction shift, it does not hold for the DEA estimator or the OLS based estimators.

Empirical Application

In this section, we apply the methodology to analyze football production. We consider data from the 2009 NFL season. The approaches developed in this article are applied to the 32 teams using regular season data. We choose wins (*W*) as the desirable outcome. Given the relatively small sample size, we choose three inputs: yards per play (*Yards*), third-down conversion success (*Third*), and penalty yards (*Penalty*). In order to capture offense and defense, we construct an index for each variable. For *Yards* and *Third*, we use the ratio of offense to defense. Because DEA maintains an assumption of monotonicity, we use the ratio of defense to offense for

Table 1. Data and Descriptive Statistics

Team	*W*	*Yards*	*Third*	*Penalty*
Arizona Cardinals	10	1.048	1.027	0.948
Atlanta Falcons	9	0.928	0.929	1.343
Baltimore Ravens	9	1.142	1.133	0.677
Buffalo Bills	6	0.959	0.637	1.075
Carolina Panthers	8	1.000	1.049	0.905
Chicago Bears	7	0.977	0.907	0.890
Cincinnati Bengals	10	0.994	1.053	0.889
Cleveland Browns	5	0.738	0.838	1.198
Dallas Cowboys	11	1.214	1.160	0.861
Denver Broncos	8	1.058	0.976	0.908
Detroit Lions	2	0.757	0.887	1.229
Green Bay Packers	11	1.213	1.306	0.865
Houston Texans	9	1.101	1.023	0.874
Indianapolis Colts	14	1.184	1.093	1.628
Jacksonville Jaguars	7	0.929	1.003	0.919
Kansas City Chiefs	4	0.814	0.717	1.231
Miami Dolphins	7	0.860	1.406	0.920
Minnesota Vikings	12	1.108	1.300	1.192
New England Patriots	10	1.085	1.177	1.050
New Orleans Saints	13	1.142	1.177	0.911
New York Giants	8	1.056	1.108	0.845
New York Jets	9	1.177	1.177	1.004
Oakland Raiders	5	0.797	0.830	0.743
Philadelphia Eagles	11	1.185	1.097	0.830
Pittsburgh Steelers	9	1.160	0.932	1.174
San Diego Chargers	13	1.123	1.099	1.395
San Francisco 49ers	8	0.996	0.813	1.278
Seattle Seahawks	5	0.871	0.854	1.156
St. Louis Rams	1	0.763	0.742	0.735
Tampa Bay Buccaneers	3	0.850	0.810	1.050
Tennessee Titans	8	1.008	1.019	0.882
Washington Redskins	4	1.008	1.002	1.124
Descriptive Statistics				
Average	8	1.008	1.009	1.023
SD	3.223	0.144	0.178	0.214
Minimum	1	0.738	0.637	0.677
Maximum	14	1.214	1.406	1.628

Penalty. As a result, an increase in any of these ratios should increase the probability of winning. Data, including descriptive statistics, are reported in Table 1.

We applied the output-oriented VRS DEA model (5). The aggregate slack was calculated to be $\sum_{j=1}^{n} S_j^* = 38.123$. With 32 teams, the resulting adjustment constant $C = \frac{1}{n}\sum_{j=1}^{n} S_j^* = 1.191$. In Table 2, we report the slack S_j^*, the adjusted slack $\tilde{S}_j^*$, and

Table 2. DEA Efficiency Results

Team		Biased TE_j	$\tilde{S}_j^*$	TE_j
Arizona Cardinals	0.53	0.95	−0.66	1.00
Atlanta Falcons	0.00	1.00	−1.19	1.00
Baltimore Ravens	0.00	1.00	−1.19	1.00
Buffalo Bills	0.00	1.00	−1.19	1.00
Carolina Panthers	2.07	0.79	0.88	0.90
Chicago Bears	1.21	0.85	0.02	0.997
Cincinnati Bengals	0.00	1.00	−1.19	100
Cleveland Browns	0.00	1.00	−1.19	100
Dallas Cowboys	1.07	0.91	−0.12	1.00
Denver Broncos	1.67	0.83	0.48	0.94
Detroit Lions	3.39	0.37	2.20	0.48
Green Bay Packers	1.21	0.90	0.02	0.999
Houston Texans	1.13	0.89	−0.06	1.00
Indianapolis Colts	0.00	1.00	−1.19	1.00
Jacksonville Jaguars	1.58	0.82	0.39	0.95
Kansas City Chiefs	0.00	1.00	−1.19	1.00
Miami Dolphins	0.00	1.00	−1.19	1.00
Minnesota Vikings	0.54	0.96	−0.65	1.00
New England Patriots	1.97	0.84	0.78	0.93
New Orleans Saints	0.00	1.00	−1.19	1.00
New York Giants	2.59	0.76	1.40	0.85
New York Jets	4.13	0.69	2.94	0.75
Oakland Raiders	0.00	1.00	−1.19	1.00
Philadelphia Eagles	0.00	1.00	−1.19	1.00
Pittsburgh Steelers	1.39	0.87	0.20	0.98
San Diego Chargers	0.00	1.00	−1.19	1.00
San Francisco 49ers	0.49	0.94	−0.71	1.00
Seattle Seahawks	2.06	0.71	0.87	0.85
St. Louis Rams	0.00	1.00	−1.19	1.00
Tampa Bay Buccaneers	3.12	0.49	1.93	0.61
Tennessee Titans	1.78	0.82	0.59	0.93
Washington Redskins	6.18	0.39	4.99	0.45

Note. All calculations by authors. The biased efficiency measure results from using the uncorrected slack.

the corresponding efficiency score. For comparison purposes, we also report the biased efficiency based on the slack S_j^*. The biased technical efficiency estimate and the adjusted technical efficiency estimates are reported for each team. Based on the new output oriented model, 5 teams (Cardinals, Cowboys, Vikings, 49ers, and Texans) that were previously identified as being inefficient have now become efficient. The average team played inefficient teams 7 times in the 16 games season. However, for the 5 teams that became efficient, they played on average 8.6 games against inefficient teams, which is more than 83% of the league. Further, the teams

Table 3. Regression Results

Variable	OLS	SAE
Intercept	−15.946* (2.491)	−17.809* (2.543)
Yards	15.246* (2.455)	15.866* (2.515)
Third	5.097* (2.001)	4.826* (2.048)
Penalty	3.366* (1.305)	4.760* (1.336)
Adjusted R^2	0.775	0.764

Note. OLS = ordinary least squares; SAE = sum of absolute errors.
* Indicates significance at the 5% level. The dependent variable is the number of wins.

that were found to be inefficient only played other inefficient teams on average 6 times in the season.

It is important to note here that this correction does not merely increase the estimated efficiency of each team; it also alters the ranking of teams based on efficiency. For example, the biased technical efficiency estimate for Green Bay (0.90) is higher than that of Houston (0.89), however, after correcting for the bias, Houston is estimated to be more efficient than Green Bay ($TE = 1$ and $TE = 0.9986$, respectively). Thus the bias in traditional estimates leads to incorrect rank ordering of teams based on biased technical efficiency estimates.

OLS and SAE regression results are reported in Table 3. In both the OLS and the SAE analysis, the three input variables *yards, third, penalty* and the intercept were all found to be statistically significant at the 95% confidence level and the adjusted R^2 values were .775 and .764, respectively. This indicates a large proportion of the variation in wins can be explained by the input variables selected. Table 4 presents the adjusted DEA technical efficiency estimates and the efficiency estimates from the OLS and SAE. The correlation between technical efficiency estimates obtained from DEA and OLS, DEA and SAE, and OLS and SAE were 0.80, 0.89, and 0.97, respectively. The high correlations between the three methods indicates considerable consensus.

Conclusion

Standard production theory assumes each production unit operates independently of the other production units. Thus, when efficiency is measured and recommended improvement strategies are advocated, these actions can be implemented and the benefits realized independent of the other production units. In sports however, where one of the outputs (wins) is strictly limited to be the number of games played, improvements in one team's efficiency necessarily implies additional losses for other teams. Recognizing this issue we have proposed modifications to the most common parametric and nonparametric approaches for estimating efficiency. Through a simple example, we illustrate how the frontier is overestimated and thus

Table 4. Regression Efficiency Results

Team	DEA TE_j	OLS TE_j	SAE TE_j
Arizona Cardinals	1.000	1.000	1.000
Atlanta Falcons	1.000	1.000	1.000
Baltimore Ravens	1.000	0.950	1.000
Buffalo Bills	1.000	1.000	1.000
Carolina Panthers	0.900	1.000	1.000
Chicago Bears	0.997	1.000	1.000
Cincinnati Bengals	1.000	1.000	1.000
Cleveland Browns	1.000	1.000	1.000
Dallas Cowboys	1.000	0.970	0.987
Denver Broncos	0.940	0.970	0.999
Detroit Lions	0.480	0.470	0.462
Green Bay Packers	0.999	0.910	0.928
Houston Texans	1.000	1.000	1.00
Indianapolis Colts	1.000	1.000	1.00
Jacksonville Jaguars	0.950	1.000	1.00
Kansas City Chiefs	1.000	0.940	0.904
Miami Dolphins	1.000	0.940	1.000
Minnesota Vikings	1.000	1.000	1.000
New England Patriots	0.930	0.990	0.992
New Orleans Saints	1.000	1.000	1.000
New York Giants	0.850	0.930	0.962
New York Jets	0.750	0.790	0.795
Oakland Raiders	1.000	1.000	1.000
Philadelphia Eagles	1.000	1.000	1.000
Pittsburgh Steelers	0.980	0.860	0.843
San Diego Chargers	1.000	1.000	1.000
San Francisco 49ers	1.000	1.000	1.000
Seattle Seahawks	0.850	0.900	0.887
St. Louis Rams	1.000	0.520	0.727
Tampa Bay Buccaneers	0.610	0.640	0.654
Tennessee Titans	0.930	1.000	1.000
Washington Redskins	0.450	0.480	0.478

Note. DEA = data envelopment analysis; OLS = ordinary least squares; SAE = sum of absolute errors. DEA efficiency results are shown for comparison.

inefficiency levels are also overestimated. We apply these insights to 2009 NFL data and find significantly different results than are given by the standard DEA or parametric methods.

In conclusion, previous work in the sport literature attempting to measure relative efficiency in sports leagues is likely to suffer from the endogeneity issue regarding efficiency estimates and thus overestimate inefficiency. In order to control for this endogeneity issues we advocate the use of the methods proposed in this article for future investigations of efficiency in sports.

Appendix

Theorem 1: Using the SAE estimator, the league production function estimated as the average of division production functions is equal to the league production function estimated using all teams in the league.

Proof: Assume a league of n teams partitioned into non-trivial sets, $i = 1, \ldots, q$.[5] Define the subset of teams belonging to set i a $j_i = 1, \ldots, \bar{\bar{i}}$ where $\bar{\bar{i}}$ is the cardinality of set i. For each set (i.e., AFC East division), a production frontier, $f_i(x_{j_i})$, will be estimated via SAE. Thus, for all i in q

$$\underset{f_i(x_{j_i}) \in \Re^n, \mu \in \Re^n_+}{\arg\ \min} \sum_{j_i} \left[y_{j_i} - f_i(\mathbf{x}_{j_i})\right].$$

Then if these solutions are averaged over sets this implies

$$\frac{\sum_i \underset{f_i(x_{j_i}) \in \Re^n, \mu \in \Re^n_+}{\arg\ \min} \sum_{j_i} \left[y_{j_i} - f_i(\mathbf{x}_{j_i})\right]}{q}$$

Since a monotonic transformation of the objective function does not change the optimal solution, the $\frac{1}{q}$ factor can be dropped. The summation over sets can be interchanged with the minimization and the result can be seen directly

$$\frac{\sum_i \underset{f_i(x_{j_i}) \in \Re^n, \mu \in \Re^n_+}{\arg\ \min} \sum_{j_i} \left[y_{j_i} - f_i(\mathbf{x}_{j_i})\right]}{q} = \underset{f_i(x_{j_i}) \in \Re^n, \mu \in \Re^n_+}{\arg\min} \sum_i \sum_{j_i} \left[y_{j_i} - f_i(\mathbf{x}_{j_i})\right],$$

where the right-hand side is now just the SAE estimated for the entire league.

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Notes

1. Only one output is used in our empirical analysis. In a later section, we extend our modeling to multiple outputs.
2. Sometimes referred to as minimum absolute deviation.

3. Carl Friedrich Gauss is credited with developing the fundamentals of the basis for least-squares analysis in 1795 at the age of 18. Legendre was the first to publish the method, however.
4. The issue of less than round robin schedule can be extended to multiple years in which no team from the current year plays the teams from prior or future years.
5. Identical arguments hold for a set of team observations gather over some time period and the sets are the individual time periods.

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