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Quantifying picker blocking in a bucket brigade order picking system

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ABSTRACT

Bucket brigade is a linear order-picking process with one loading station and one unloading station. Here we model and quantify picker blocking in bucket brigade order picking systems (OPSs). A bucket brigade improves throughput and reduces variability in OPSs. However, each order picking trip fills different orders and creates workload variation per order. We show that bucket brigade order picking experiences picker blocking when there is a workload imbalance per pick face. We derive a closed-form solution to quantify the level of blocking for two extreme walk speed cases. Additional simulation comparisons validate the picker blocking model which includes backward walk and hand-off delays. We identify the relationship between picker blocking in bucket brigade OPSs and picker blocking in a circular-aisle abstraction of OPSs in which backward walk and hand-off delays as well as forward walk speed are considered. Our analytical model and simulations show that aggregating orders into batches smoothes the workload variation by pooling the randomness of picks in each order and that slowest-to-fastest picker sequencing modulates picker blocking between two pickers, *i.e.*, the interaction between neighboring pickers.

1. Introduction

Distribution centers (DC) receive products from suppliers and fulfill orders for customers. Order picking refers to the process of retrieving items from storage locations to fulfill customer orders. Tompkins et al. (2003) report that OPSs on average consume 55 percent of a retailer DC's operational cost. This cost will likely be higher for an OPS under pressure to absorb demand variability from market fluctuation (Hong et al., 2012b) and to resolve skill discrepancy from frequent workforce changes (Bartholdi and Eisenstein, 1996a). A bucket brigade strategy is useful for order picking both in warehouses and many types of manufacturing processes. Its characteristics of high throughput and a self-organizing property allow workforces to be organized with a minimal level of managerial planning and oversight. In this paper, we refer to the combination of flow-rack shelving (Fig. 1(a)) and the bucket brigade strategy discussed in Bartholdi and Eisenstein (1996a) as a bucket brigade order picking system (OPS). We define the bucket brigade OPS as a linear order-picking process with one loading station and one unloading station (Fig. 1(b)). Pickers travel through an aisle to retrieve items from shelves and place them in a bin (or tote) on a conveyor.

The picking area is divided into "zones" in which a picker picks a batch (or order). However, unlike other types of zone picking, the boundaries between the zones are continuously updated to maintain high utilization of the pickers and to minimize the work in process (WIP). The picker in the first zone, the most upstream picker, picks an item and places it in the tote assigned to a particular batch (Fig. 2(a)). Then the upstream picker moves to the next pick face to continue processing the batch (Fig. 2(b)) by picking at subsequent pick faces until meeting a downstream picker who has no assigned tote. At this point, the upstream picker hands off the current tote and returns to the loading station, or meets another upstream picker. If this individual is not the most upstream picker, after handing off the tote, the upstream picker moves backwards to meet a picker further upstream (Fig. 2(c) and (d)). Upon finally meeting an upstream picker, the now-downstream picker takes over the upstream picker's tote and walks forward until either meeting another downstream picker without a tote, or reaching the unloading station (Fig. 2(b) and (d)). The last downstream picker releases the completed batch to the unloading station and moves backward to take over a new tote (Fig. 2(c)). This so-called dynamic pick-and-pass process eliminates the need for workload balancing and minimizes WIP (Bartholdi and Eisenstein, 1996a).

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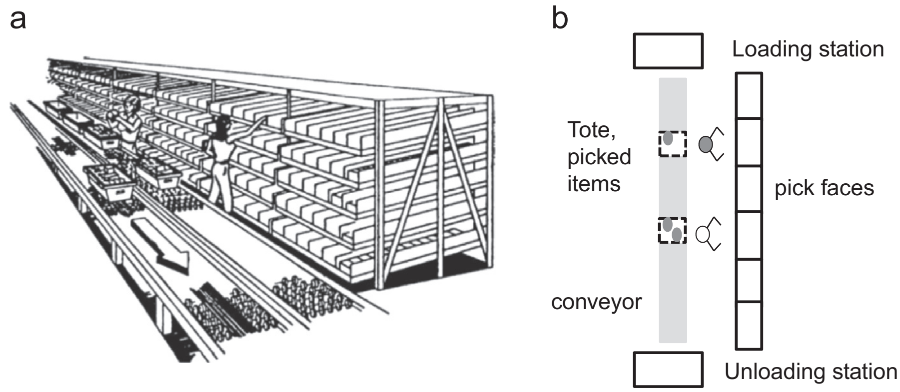

Fig. 1. A flow-rack OPS: (a) physical layout; and (b) top view.

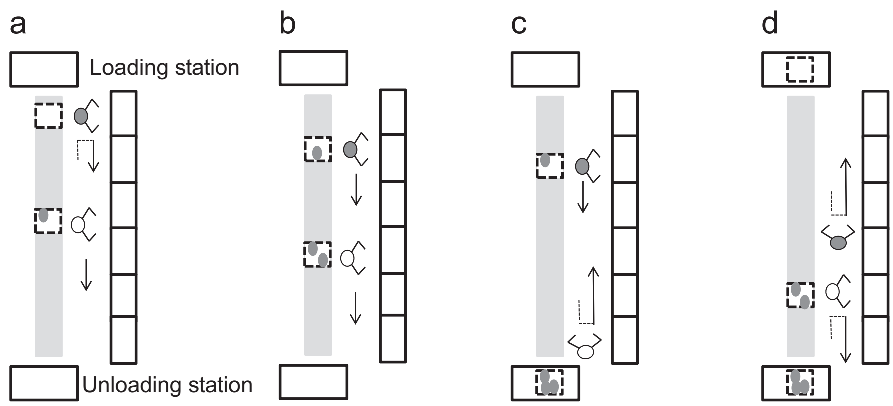

Fig. 2. A series of bucket brigade order picking operations: (a) an upstream picker takes a tote from a loading station and a downstream picker travels forward while picking; (b) both pickers travel forward and pick items; (c) a downstream picker unloads a tote and travels backward; and (d) a downstream picker takes over a tote and an upstream picker hands off a tote.

In a bucket brigade OPS, randomness and the imbalanced workload between pickers cause picker blocking. For example, an upstream picker in Fig. 2(b) attempts to move forward to the next pick face, which can be occupied by a busy downstream picker (Fig. 3(a)). In this example, *picker blocking* occurs when the upstream picker cannot hand off the current batch to the downstream picker, because the downstream picker is currently picking, and the upstream picker cannot pass the downstream picker, because the pickers' sequence must be maintained. In addition, pickers stand idle when the hand-off process is synchronized improperly. If an upstream picker in Fig. 2(d) is picking when a downstream picker encounters the upstream picker (Fig. 3(b)), the downstream picker must wait until the upstream picker completes the pick and *hand-off delay* occurs.

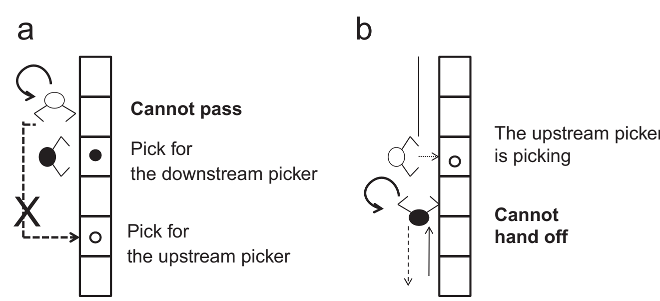

Fig. 3. Delay situations in bucket brigade order picking: (a) picker blocking; and (b) hand-off delay.

Blocking delays and the resulting hand-off delays occur (Bartholdi and Eisenstein, 1996a), when pick requirements are random over pick locations. Picker blocking in bucket brigade order picking has received little attention in the literature, with a few notable exceptions (Armbruster and Gel, 2006; Armbruster et al., 2007; Bartholdi and Eisenstein, 1996b, 2005; Bartholdi et al., 2001); however, this research has focused on operational rules or conditions that lead to reasonable overall operational performance in diverse settings. A clearly defined analytical model for upstream blocking delays is needed and we provide one such model in this paper.

We develop a model for picker blocking with unique characteristics specific to bucket brigade order picking by measuring and tracking the distance between order pickers. Further, we analytically investigate two extreme conditions of very-fast and very-slow forward walking speeds and discuss simulation studies to identify picker blocking patterns for typical forward and backward walk speeds including hand-off delays. Our simulation study also analyzes how picker blocking in a bucket brigade OPS occurs due to variations in the hand-off times between pickers, backward work speed, batch picking, and workforce staffing. Finally, the simulations compare bucket brigade and a circular-aisle OPSs abstraction in terms of picker blocking and the impacts on workload balance.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature and describes the picker blocking issues. Section 3 analyzes picker blocking in bucket brigade order picking. Section 4 introduces analytical models for picker blocking. Section 5 details the picker blocking *via* simulation studies. Section 6 concludes.

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2. Literature review

2.1. Bucket brigades

Bartholdi and Eisenstein (1996b) introduce the bucket brigade workforce management method based on three assumptions: workers travel with instantaneous walk speed, each worker's capability is distinctive, and workload is uniformly and randomly distributed. The highest throughput is obtained when pickers are sequenced with the slowest worker in the location most upstream and the fastest worker in the location most downstream. Further, Bartholdi and Eisenstein (1996b) show that blocking delay is minimized when large capability differences exist among pickers. Bartholdi and Eisenstein (1996a) suggest that cooperation between neighboring workers can also reduce blocking delay *via* smoother hand-offs. Bartholdi et al. (2001) relax the assumption that the workload is uniform over the pick area and conclude that bucket brigades are still advantageous and self-balancing.

In a manufacturing example, Bartholdi and Eisenstein (2005) analyze an assembly line with a finite walk speed and a lengthy return trip after handing off a workload. Under these assumptions, they find a considerable loss of productivity due to walk-back time and hand-off delay. Armbruster et al. (2007) consider a situation where pickers learn and improve workspeed; in their alternative model, resequencing to maintain slowest-to-fastest ordering improves performance. Koo (2009) shows that picker blocking and hand-off delay reduce bucket brigade OPS productivity when pickers have identical capabilities. To improve productivity, each picker's picking area is constrained by defining a downstream boundary where upstream pickers are allowed to leave totes if a downstream picker is not available. While this approach decreases delay, it increases WIP.

2.2. Picker blocking in narrow-aisle order picking

Our research builds on previous analysis of picker blocking in narrow-aisle OPSs. For example, Skufca (2005) presents a k-picker congestion model of a no-passing system in the case of infinite walk speed. Gue et al. (2006) address two-picker congestion models of a parallel-aisle pick area approximated by a circular no-passing system considering unit and infinite walk speeds. In the unit walk speed case, the unit walk time to pass a pick face is identical to the unit pick time. Gue et al. (2006) also conduct simulation experiments to investigate picker behavior under more practical walk speed assumptions, by identifying the effects of pick density on narrow-aisle order picking performance under the single-pick assumption where a picker has at most one pick at a pick face. They conclude that a batch picking strategy in narrow-aisle OPSs is advantageous when the pick density is either very low or very high. Parikh and Meller (2010) and Hong et al. (2013) show that the variation in pick density can be as important as the level of pick density in determining the amount of blocking in a circular-aisle OPS abstraction. Hong et al. (2012b) confirm that batching strategies can reduce picker blocking regardless of the pick density.

Hong et al. (2012a) develop a narrow-aisle indexed batching model (IBM) that generates batches to control picker blocking in an OPS with multiple narrow-aisles where passing in an aisle is not possible. A batch index represents the batches' release sequence; the model assigns orders to indexed batches and determines the retrieval routes for each batch.

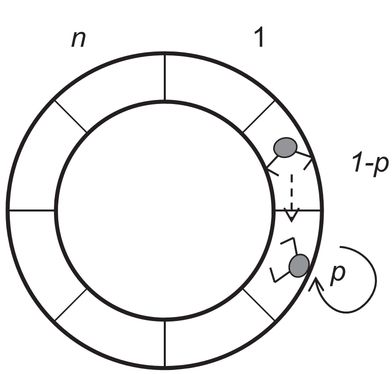

Fig. 4. A typical no-passing circular passage system: a circular-aisle warehouse.

2.3. Issues

Many studies of the differences in blocking and throughout for bucket brigade OPSs do not quantify picker blocking or explain the effects of bucket brigade strategies on picker blocking. The work most closely related to our paper, Koo (2009), investigates performance *via* simulation and suggests changing the bucket brigade strategy. Thus, we are motivated to analytically understand and model picker blocking within a bucket brigade OPS, to describe the interaction between pickers at different pick locations, and to explore the effects of bucket brigade strategies.

3. Picker blocking mechanism

We begin by discussing a model of picker blocking which uses a circular-aisle abstraction and explaining the typical release mechanism of a new batch. Then we show the equivalence under specific situations of the picker blocking models between a bucket brigade OPS and a circular-aisle abstraction.

3.1. Picker blocking in a circular-aisle abstraction

A circular-aisle abstraction is useful to understand picker blocking phenomena (Gue et al., 2006; Hong et al., 2013; Parikh and Meller, 2010; Skufca, 2005). Fig. 4 shows a circular-aisle abstraction of a warehouse composed of linked n pick faces ($n > 2$) where pickers travel in a clockwise direction. A stochastic model determines the number of picks at a pick face. Pickers continue to pick at the same pick face with probability p. They spend unit time to conduct one pick. When no pick is chosen ($1-p$), they move to the next pick face with time m, *i.e.*, the walk speed between two pick faces is $1/m$. After a pick, the picker's probability of walking is independent of his/her previous action. Thus the probability of ν picks at a particular pick face is p^{ν}. Blocking occurs under a no-passing restriction. If the downstream picker is picking, the upstream picker cannot pass and thus remains idle. When the blocking picker completes the picking operation and a blocking situation is released, each picker moves to his/her next pick column simultaneously.

The productivity loss by picker blocking is characterized by the *ratio of time spent to pick* to *time spent at a stop*. This ratio can be less than one if picker blocking occurs. Gue et al. (2006) introduce a throughput model for an order picking system with k pickers when one pick at most is made at a stop. To reflect a batch picking situation, we generalize their model as Eq. (1). When each picker is blocked $b(k)$ fraction of the time, $0 \le b(k) \le 1$, the throughput is

$$\lambda(k) = k \cdot \left[\frac{E[p]t_p}{E[p]t_p + t_w}\right](1 - b(k)) \tag{1}$$

where $E[p]$ represents the expected number of picks at a stop. Time to pick (t_p) includes the time spent picking an item and time to walk (t_w) represents the average time to walk past a pick face (location).

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3.2. Throughput of a bucket brigade OPS under picker blocking and hand-off delays

Similar to the circular-abstraction, we express the productivity loss by picker blocking in a bucket brigade OPS as the *ratio of time spent to pick* to *time spent at a stop*. Using the throughput model of Gue et al. (2006) for an order picking system with k pickers in a single-pick situation, we generalize their model as Eq. (2). When each picker is idle, $h(k)$ represents the time per pick face due to hand-off delays, $0 \le h(k) \le max[p]$, and $max[p]$ represents the maximum number of picks at a stop; the throughput is

$$\lambda(k) = k \cdot \left[\frac{E[p]t_p}{E[p]t_p + t_{fw} + t_{bw} + h(k)}\right](1 - b(k)) \quad (2)$$

where time to walk (t_{fw}) represents the average time to forward walk past a pick face (location) and time to walk (t_{bw}) represents the average time to backward walk past a pick face (location).

3.3. Picker blocking in bucket brigade order picking

Assume that there is no hand-off delay and that backward walk speed (empty travel walking speed) is instantaneous, similar to Bartholdi and Eisenstein (1996a). In addition, assume that k pickers have identical pick performance and walk speed. Interestingly, with infinite backward walk speed and no hand-off delays, the circular-aisle abstraction picking system can be used to characterize a bucket brigade OPS in terms of picker blocking. Further, the same picker blocking model can be used for both analyses.

The equivalence is easily shown by replacing “pickers” in the circular-aisle abstraction with “batches” in the bucket brigade OPS. Recall that in bucket brigade order picking, picker blocking occurs when an upstream batch cannot proceed because a downstream batch has a pick at the pick face where the next item in the upstream batch is located (or a location between the upstream batch's current location and the next pick location) and the upstream batch is prevented from moving forward. When the upstream batch stays at the current pick face, a hand-off delay occurs. A rigorous proof follows.

Theorem 1. *When the backward walk time is instantaneous and the hand-off time is zero, the picker blocking model of bucket brigade order picking is equivalent to the picker blocking model of the circular-aisle abstraction.*

Proof. When the batch most downstream is completed, it disappears from the system, other batches in the system are handed off to the next pickers, and a new batch is released. The completion, backward walks, and hand-offs occur instantaneously and result in the release of a new batch. This proof shows that: 1) the order picking mechanisms in the bucket brigades and the circular-aisle abstractions are equivalent until a batch is completed; and 2) the completion of a batch does not impact any locations and times of current batches.

1) Before completion of the batch most downstream
   Without loss of generality, before completion of a batch, the two models follow the same procedure. For example, Fig. 5(a) shows batches i, $i+1$, $i+2$, and $i+3$ in a circular-aisle abstraction, and Fig. 5(b) shows a bucket brigade order picking situation. The unidirectional travel in the bucket brigade is analogous to the clockwise travel in the circular abstraction. Thus, until batch i (b_i) is completed, the two systems face the same situations of picker blocking.
2) Completion of the batch most downstream and occurrence of hand-off

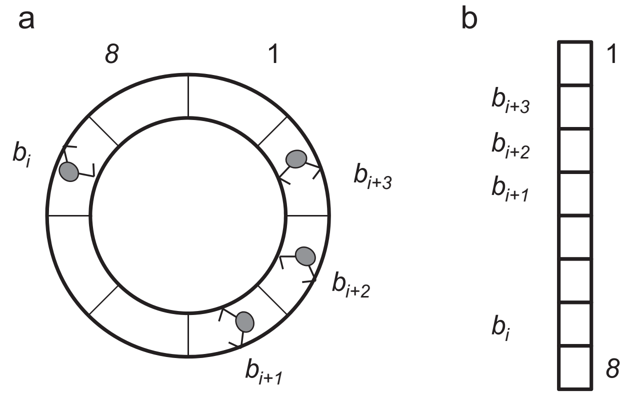

Fig. 5. In both models, four pickers process four batches. Two pickers (pickers 3 and 4) may have a chance of blocking depending on items in batches $i+2$ and $i+3$ (the number of pick faces=8, the number of pickers=4): (a) a circular-aisle abstraction; and (b) a bucket brigade OPS.

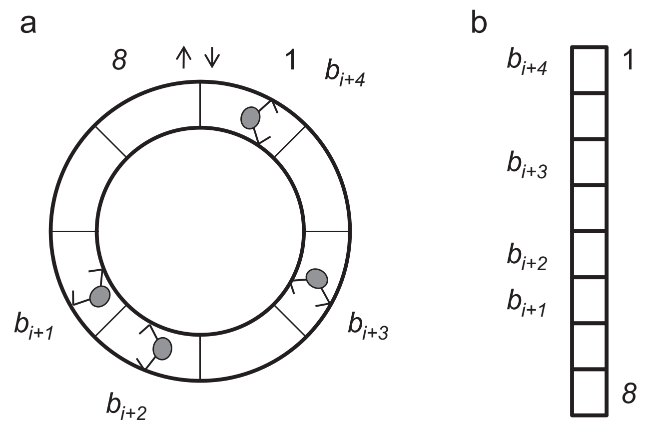

Fig. 6. A completion and release example. Both models release batch $i+4$ at the same time starting from pick face 1 (the number of pick faces=8, the number of pickers=4): (a) a circular-aisle abstraction; and (b) a bucket brigade OPS.

   Since batch i has been completed, the chain reaction discussed in Fig. 6 arises. Due to the infinite backward walk speed and the zero hand-off delay, all batches will be handed off at the same time. Batch $i+k$ enters the system (*i.e.*, the first pick face) and its release time is identical to the completion time of batch i. The picker assignments of batches $i+1$, $i+2$, …, $i+k-1$ are changed from 2,3,…,k to 1,…,$k-1$. Picker k captures batch $i+k$. During this shift, there is no blocking. Then, recursively, case 1) above repeats. In the circular-aisle abstraction, the release location of a new batch is the first pick face and the release time of a new batch is the completion time of the kth previous batch. Thus, the two systems release a new batch to the same location at the same time when the backward walk speed is infinite and the hand-off delay is negligible (see Fig. 6).

From arguments 1 and 2, the two systems are identical in steady state. While the initialization and finalization stages are beyond the scope of this paper, technically, the two models can begin with the same procedure if both models start together from the loading station, and both models can end if they do not allow any hand-off after the last batch enters the system.

End of proof.

The literature reports similar results, although the equivalence between the circular-aisle abstraction and bucket brigade picker blocking models has not been identified previously. For example, Gue et al. (2006) and Bartholdi and Eisenstein (1996b) find that batch picking results in less picker blocking and more productivity. While the relationship between the two models of order picking is clear, we believe that a rigorous definition of the relationship between the two models provides the basis for the main results described in the following sections.

4. Analytical models

This section investigates the functional relationship between workload (*i.e.*, pick density) and picker blocking in a two-picker bucket brigade OPS. We quantify picker blocking for extreme walk speed assumptions using closed-form expressions and extend the results to more realistic walk speeds through simulations. We first analyze picker blocking under the assumption of infinite walk speed, a popular assumption in the literature (Bartholdi and Eisenstein, 1996a).

4.1. Problem definition

We introduce an abstracted bucket brigade model with two pickers (Fig. 7). The bucket brigade OPS consists of n pick faces. The order pickers take a no-passing traversal route, meaning that they travel through the OPS and pick in the forward trip. Upon meeting a downstream picker, they wait until the downstream picker hands off the loaded tote, and then a hand-off occurs and the upstream picker takes the backwards trip. The downstream picker arriving at the last pick face moves backward until meeting an upstream picker. We assume that at a pick face, the pickers pick with a probability p. The probability of walking past a pick face is $q=1-p$, and the states are picking, walking, or standing idle due to blocking. Pick time is constant regardless of pick face characteristics such as shelf height. The pick time of one sku regardless of quantity, pt, the forward walk time between two pick faces, ft, and the backward walk time between two pick faces, bt, are all deterministic. The loading and unloading times are independent of the picker blocking or other operational issues and thus we treat them as zero.

D_t denotes the distance between two pickers at time t. The distance $d \in D_t$ ranges from 0 to n, where 0 and n represent a downstream picker blocking an upstream picker. Measuring D_t follows the distance definition in Fig. 8. The physical layout of the distance between two pickers is shown in Fig. 8(a). After the pickers begin to pick, the distance between them is defined as $D_t^{down}=(\textit{downstream picker position}-\textit{upstream picker position})$. Until the downstream picker reaches position n, this measurement is valid, but when the downstream picker arrives at position n and starts to move upstream, the distance is defined as $D_t^{up}=n-(\textit{downstream picker position}-\textit{upstream picker position})$. The calculation of the D_t value switches between D_t^{down} and D_t^{up} whenever

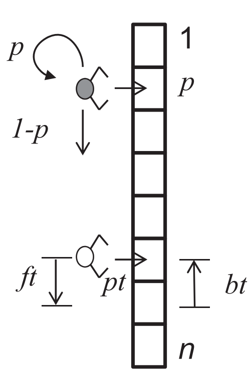

Fig. 7. Abstracted model.

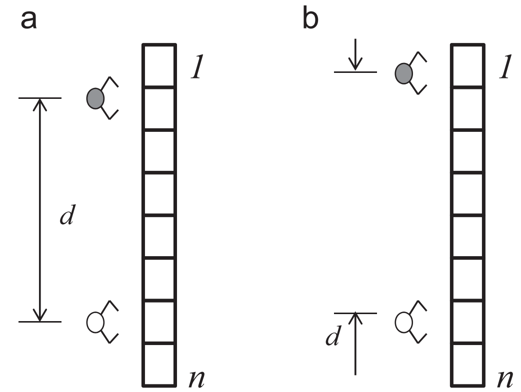

Fig. 8. Two distance measurements: (a) the downstream picker takes an odd-numbered tote; and (b) the downstream picker takes an even-numbered tote.

the downstream picker unloads a tote at an unloading station or hands off.

When the downstream picker unloads at position n at time $t-1$, a hand-off occurs at time t (Fig. 8(a)), since, instantaneously, the downstream picker unloads the tote at the unloading station and walks backward to the upstream picker. Because the downstream picker is now idle, the upstream picker yields the current tote and moves to position 1 to take a new tote from the loading station. The new positions of the two pickers at time t are shown in Fig. 8(b). In this case, we measure the distance between the two pickers as $n-(\textit{downstream picker position}-\textit{upstream picker position})$. Restated, if the distance at time $t-1$ is r, the distance at time t is $n-(n-(r+1))=r+1$.

Note how our distance definition smoothly connects two neighboring distance states without changing the probability of picker blocking between the two pickers. Another way to express the distance definition is that when the downstream picker takes odd-numbered totes (1, 3, 5, …), the distance becomes (*downstream picker position* − *upstream picker position*), and when the downstream picker takes even-numbered totes (2, 4, 6, …), the distance becomes $n-(\textit{downstream picker position}-\textit{upstream picker position})$.

We derive the percentage of time blocked, denoted as $b_{pt:ft:bt}(n)$, where pt:ft:bt represents the pick:forward walk:backward walk time ratio as a performance measure and n represents the number of pick faces in the system. We identify analytical models over two restrictive cases: 1) forward walk speed is equal to unit pick time per pick face and backward walk speed is infinite (pt:ft:bt=1:1:0); and 2) forward and backward walk speeds are infinite (pt:ft:bt=1:0:0). We assume that the pick time is proportional to the number of picks.

4.2. Extremely slow forward walks and instantaneous backward walks (pt:ft:bt=1:1:0)

We utilize a Markov property for determining distances between two pickers consistent with Hong et al. (2013). A Markov chain is introduced by defining state $S_t=0$ (*block* in the downstream picker's odd-numbered trip), state $S_t=n$ (*block* in the downstream picker's even-numbered trip), and states [1, 2, …, $n-1$] given by $S_t=D_t$ according to the distance definition discussed above (see Fig. 8). All states can be summarized by the vector [0, 1, 2, …, $n-1$, n]. We distinguish four transition cases and show that the current state S_t is determined by S_{t-1} and that a Markov property is valid.

a) Transition probabilities between unblocked states

If both pickers pick ($p*p$) or walk ($q*q$), the current distance

(D_t) does not change at $t+1$. If a hand-off occurs, the current distance is the same according to the assumption of D_t regardless of each picker's position. However, when picker 1 (who is a downstream picker when the downstream picker picks an odd-numbered batch and an upstream picker when the downstream picker picks an even-numbered batch) picks while picker 2 walks ($p*q$), the distance decreases by 1. When picker 1 walks while picker 2 picks ($q*p$), the distance increases by 1.

b) Transition probabilities from an unblocked state to a blocked state
When the distance between picker 1 and picker 2 is 1, a blocked state can arise if picker 1 picks (with probability p), and picker 2 walks (with probability q). *Vice versa*, when the distance between picker 1 and picker 2 is $n-1$, the current state becomes a blocked state if picker 1 walks (with probability q) and picker 2 picks (with probability p).
When the distance is $n-1$ and picker 1 is located at pick face n, if picker 1 walks to the unloading station and picker 2 is located at pick face 1, the occurring hand-off leads to picker blocking (Fig. 9(a)). Then, according to D_t, the downstream picker (picker 2) occupies pick face 1, the upstream picker (picker 1) is blocked at the loading station, and the same probability relationship holds. Because it is a blocking status, the distance is n. Similarly, when the distance is 1 and picker 1 is located at pick face 1, if picker 1 stays at pick face 1 and picker 2 walks at pick face n, the occurring hand-off leads to picker blocking (Fig. 9(b)).

c) Transition probabilities from a blocked state to an unblocked state
If picker 2 blocks picker 1, picker 1 must wait for picker 2 to walk (with probability q) to exit a blocked state and *vice versa*. When the two pickers are located at pick faces $n-1$ and n, the forward move at pick face n leads to a transition to an unblocked state.

d) Transition probabilities between blocked states
When the current state is blocked, a pick can occur with probability p. The blocking status remains, *i.e.*, a blocked state transits to a blocked state with probability p, but hand-off delays do not affect these states.

When multiple-picks are allowed, a Markov property of distance holds over pick probability p. Fig. 10 depicts the transition diagram characterizing the state space and transition probabilities. This diagram is the same as the diagram in Hong et al. (2013), which was derived for the parallel aisle OPS. Note that the proposed discrete-time Markov chain of picker blocking for multiple-picks with a pick:forward walk:backward walk times = 1:1:0 is not conditioned on the pickers' hand-off operation.

The stationary probabilities for the Markov chain model are the same as Hong et al. (2013). The blocking probability per picker in a blocked state is

$$b_{1:1:0}(n)=\frac{p}{2p+n-1} \tag{3}$$

Here, only the upstream picker experiences picker blocking and the blocking probability of the upstream picker is equal to $2p/(2p+n-1)$. We validate the analytical model in Eq. (3) *via* a comparison with simulation models for four different layouts ($n=10$, 20, 50, and 100). The relative error gap between the analytical models and the four simulation results shows, on average, 0.52%, with a minimum gap of 0.00% and a maximum gap of 5.6% (see Appendix A).

4.3. Instantaneous forward/backward walks (pt:ft:bt=1:0:0)

We now show how the current distance shortens or lengthens for the 1:0:0 model of a bucket brigade OPS. All states are summarized by the vector $[0, 1, 2, \ldots, n-1, n]$, which derives from the previous unit-walk time model. Assume that random variables X_t^1 and X_t^2 represent the number of pick faces passed in time t by picker 1 and picker 2, respectively. $Y_t=Diff\,(X_t^1, X_t^2)$ represents the difference in walks between the two pickers. The probability density function of Y_t, $g(y)$ (see Hong et al. (2013) for detail) is

$$g(y)=P(Y_t=y)=p^2q^{-y}\frac{1}{(1-q)(1+q)}=\frac{pq^{|y|}}{1+q}\quad \text{for} -\infty<y<\infty. \tag{4}$$

If we assume that the distance at the previous state is $D_{t-1}=r$, the actual change in distance will be bounded by the physical blocking phenomenon and the value r, which gives two transition cases:

a) Transition probabilities to unblocked states
In this case, the distribution function (4) is used directly. Thus, the change, given r, is from 1 to $n-1$:

$$P(Y_t=y)=g(y)=\frac{pq^{|y|}}{1+q}\quad \text{for}\ \ -1-r<y<n-1-r,\quad r=0,\ldots,n$$

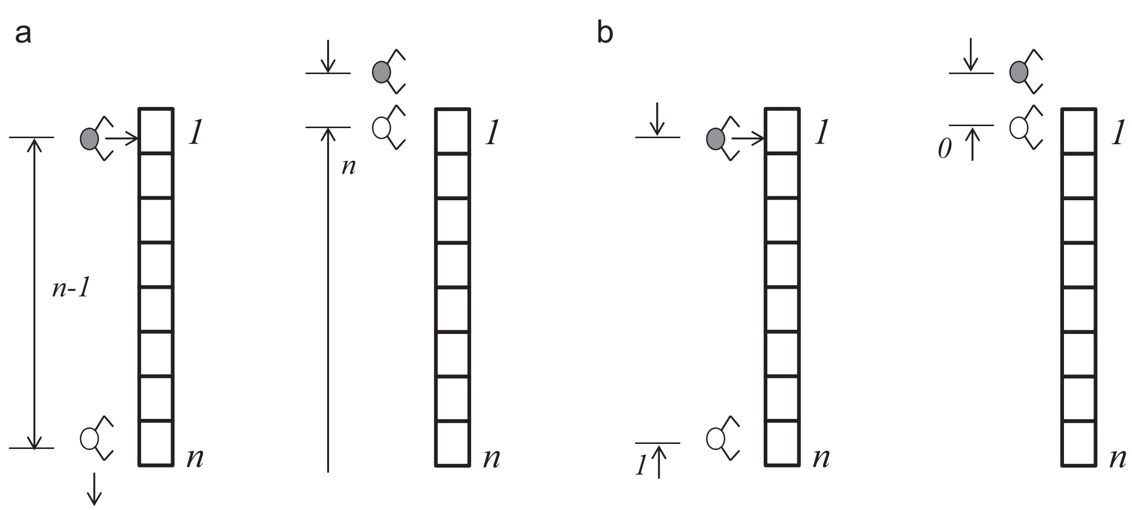

Fig. 9. Picker blocking at the last pick face when: (a) the downstream picker takes an odd-numbered tote; and (b) the downstream picker takes an even-numbered tote.

Note that r is 0 or n when a picker is blocked. Denote L^1_{t-1}, L^2_{t-1} the location of picker 1 and picker 2 at $t-1$. The locations of two pickers at t are $L^1_{t-1}+X^1_t$ and $L^2_{t-1}+X^2_t$.

i) Odd-numbered tote
When there is no hand-off between $t-1$ and t, the change in distance (d) equals $[L^1_{t-1}+X^1_t-(L^2_{t-1}+X^2_t)]-r=[r+y]-r=y$. If a hand-off occurs, the change in distance (d) equals $[n-((L^2_{t-1}+X^2_t)-(L^1_{t-1}+X^1_t-n))]-r=[n-(-r-y+n)]-r=y$.

ii) Even-numbered tote
When there is no hand-off between $t-1$ and t, the change in distance (d) equals $n-[(L^2_{t-1}+X^2_t)-(L^1_{t-1}+X^1_t)]-r=[r+y]-r=y$. If a hand-off occurs, the change in distance (d) equals $[L^1_{t-1}+X^1_t-(L^2_{t-1}+X^2_t)]-r=[r+y]-r=y$.

b) Transition probabilities to blocked states
The next step calculates the probability of blocking events by summing all cases of blocking (1 or $n-1$). We note that blocking will occur at state 0 if $Y_t \le -r$. $g(y)$ is symmetric. The probability of blocking is calculated as

$$P(Y_t \ge r)=\sum_{y=r}^{\infty}\frac{pq^{|y|}}{1+q}=\frac{p}{1+q}q^r\frac{1}{1-q}=\frac{q^r}{1+q}$$

$$P(Y_t \ge n-r)=\frac{q^{n-r}}{1+q}$$

We also need to check the transition to blocked states after a hand-off. Because the blocked states do not change the downstream picker's position, there is one hand-off at most and the probability function still holds. The two situations appear in the transition diagrams in Fig. 11. This model is identical to the probability model of the circular-aisle abstraction described in Hong et al. (2013) (see Hong et al. (2013) for details of the calculation).

The blocking probability per picker in one of the blocked states is

$$b_{1:0:0}(2)=\frac{1}{2+(n-1)p} \tag{5}$$

Consistently, the blocking probability of the upstream picker is equal to $2/(2+(n-1)p)$ and the error gap between the analytical models and the four simulation results is, on average, 0.19%, with a minimum gap of 0.00% and a maximum gap of 1.22% (see Appendix A).

The analytical models are identical to the models of the circular-aisle abstraction OPS with two pickers. Still, several questions remain. First, realistic forward walk time between pick faces is not instantaneous, or one unit of time, as assumed in the analytical model. In addition, unlike the circular-aisle order picking model, bucket brigade order picking involves hand-off delays as well as blocking delays and backward walking time. Both hand-off delay and backward walk impact picker blocking.

5. Simulation models

This section describes the simulation models used to evaluate the operation of bucket-brigade OPSs under specific range of operating conditions. We show that the analytical models bound the feasible range of walk speeds by considering a very slow walk speed and hand-off delays. Additionally, we measure the impacts of backward walk speed on picker blocking and limited number of picks per stop. Finally, we include simulations with five pickers and variably skilled pickers under the assumption of limited number of picks per trip. For each simulation, we consider pick density value on the range of 0.01–0.99 with the interval=0.01 with 20 runs per instance.

5.1. Non-extreme forward walk speed and instantaneous backward walks (impacts of hand-off delay)

We examine a common forward walk speed in bucket brigade order picking. Note that the backward walk speed is still infinite, or at least much greater than the forward walk speed (Bartholdi and Eisenstein, 1996a). The resulting hand-off delay as a function of pick density is shown in Fig. 12. Comparing a bucket brigade system with a circular-aisle abstraction for the same pick to forward walk time ratios shows that less picker blocking in the bucket brigade system than in the circular-aisle abstraction due to the hand-off delay. Simply stated, part of the delay that would be attributed to blocking in the circular-aisle abstraction is now attributed to the hand-off delays in the bucket brigade system.

Recall that when forward walk speed is not infinite or is not unit walk speed, a hand-off delay occurs and the starting time of a

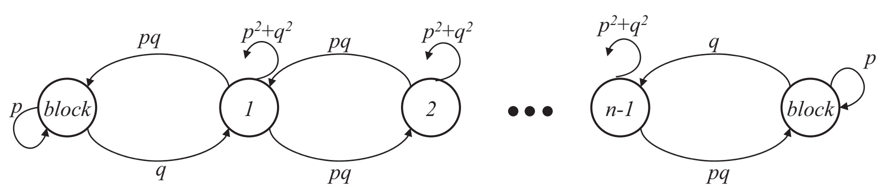

Fig. 10. State space and transitions for the Markov chain model when pickers take extremely slow forward walks and instantaneous backward walks (pt:ft:bt=1:1:0).

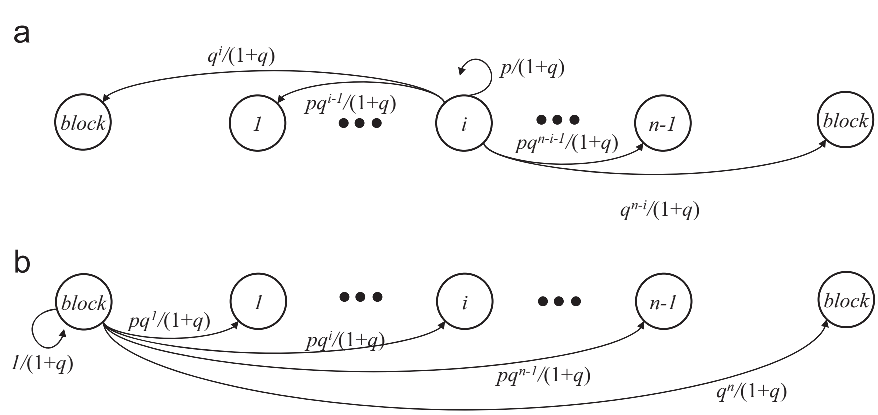

Fig. 11. State spaces and transitions for the Markov chain model when the forward and backward travel times are infinite: (a) unblocked case; and (b) blocked case.

new batch is delayed. A hand-off delay occurs as two pickers become asynchronous. Fundamentally, the infinite walk speed model causes pickers to confront each other in unit pick time. As long as two pickers begin at the same unit time, they are synchronized in a unit interval time. In the unit walk speed case, a similar situation is expected. However, in general, the walk speed may be between infinite and unit walk speed in an order picking scenario (Gue et al., 2006). In this case, the timing of confrontation of the two pickers can be asynchronous and cause a hand-off delay. Thus, picker blocking probability decreases. The example in Fig. 13 illustrates a hand-off delay of picker 1 when picker 2 processes the second item of batch 2 (B_2^2, where the superscript represents the batch number and the subscript represents the items in the batch). Picker 1 has completed the last two picks of batch 1 (B_5^1 and B_6^1) and unloaded the collected batch, and is idle at the pick face next to picker 2. The idle time duration is Y_1 $(=A_2-S_1)$ when picker 1 arrives at time S_1 and picker 2 processes the second item of batch 2 (B_2^2) at time A_2.

Next, assume that hand-off delays can occur and that backward walk speed (empty travel walk speed) is still instantaneous. Denote $E[HO]$ as the expected hand-off time per trip; by assumption, bt is 0. Denote the time blocked probability in the n-pick face circular aisle as $B_{pt:ft}(n)$. The picker blocking in the n-pick face bucket brigade OPS can be approximated as:

Corollary 1. When the backward walk time is instantaneous

$$b_{pt:ft:0}(n) \approx B_{pt:ft+E[HO]/n}(n)$$

Proof. Each trip has one hand-off delay when there are two pickers. The hand-off delay impacts the downstream picker's trip and the unproductive time impacts the walk time. When the expected hand-off delay, $E[\mathrm{HO}]$, is positive the trip lengthens by $E[HO]$. Thus, the average walk time between two pick faces becomes $ft+E[\mathrm{HO}]/n$.

End of proof.

Fig. 14 compares $b_{pt:ft:0}(n)$ to $B_{pt:ft+E[HO]/n}(n)$, where Fig. 14 (a) represents the picker blocking ($b_{pt:ft:0}(n)$) and Fig. 14 (b) represents the circular-aisle simulation ($B_{pt:ft+E[HO]/n}(n)$). Here, Corollary 1 is almost true when the walk speed is slow, but as it becomes faster, the error gap, on average, is 0.66%, with a maximum gap of 14.69% and a minimum gap of 0.00%. Corollary 1 also explains the trade-offs between hand-off and blocking delay in bucket brigades. Even though one might expect the hand-off delay to worsen the blocking delay, we find the productivity loss due to hand-off delay is partially compensated by the productivity gain by reducing blocking delay because a blocking model is equivalent to a blocking model with a slower walk speed.

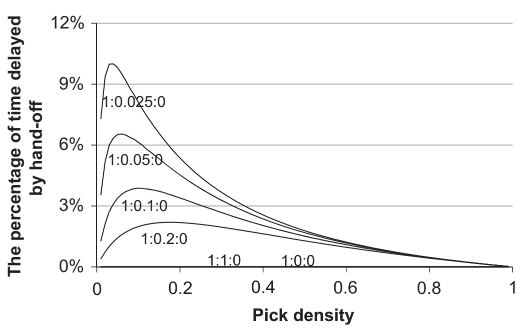

Fig. 12. Percentage of time delayed by hand-off in bucket brigade OPS (two-picker, 20 pick faces).

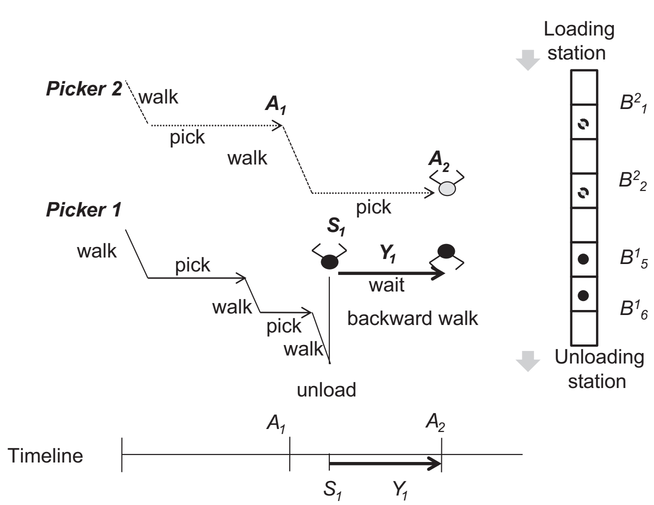

Fig. 13. An example of hand-off when the forward walk speed is not extreme.

5.2. Finite backward walk speed

Now consider when a picker in a bucket brigade order picking returns to pick a new batch from the preceding picker or from the loading station. Assume that the picker's return walk speed is finite. Fig. 15 analyzes the impacts by hand-off delays and backward walk speed over the workload., *i.e.*, backward walk speed has significant impacts when the workload is low and the impacts lessen as the workload increases.

Corollary 1′. In general, the picker blocking in the n-pick face bucket brigade OPS is

$$b_{pt:ft:bt}(n) \approx B_{pt:ft+bt+E[HO]/n}(n)$$

Proof. Extending Corollary 1, each trip walks n pick faces backward. If a picker moves forward one pick face, later the picker must move back to the pick face with backward walk time bt. Thus, the expected walk time between two pick faces becomes $ft+bt+E[\mathrm{HO}]/n$.

End of proof.

Fig. 16 shows the results of simulating Corollary 1′. Note that the values overestimate delays when the pick density is lower and underestimate when the pick density is high. The error gap, on average, is 2.64%, with a maximum gap of 30.93% and a minimum gap of 0.00%. In general, a low pick density leads to a relative long time interval with no blocking, whereas in a high pick density case, the backward walk skews pickers' walk speeds and more picker blocking occurs.

The backward walk impacts the blocking delay. Intuitively, moving backward reduces the distance between the upstream and downstream pickers. A faster backward moving speed would reduce the time to blocking. Given the forward walk speed, the earlier blocking inevitably increases blocking chances and results in more delay. Corollary 1′ explains that the circular-aisle abstraction blocking model, which has no backward moving, is equivalent to the bucket brigade blocking model when the forward walk speed of the circular-aisle abstraction blocking model is slower than the forward walk speed of the bucket brigade blocking model. Therefore, the adjusted circular-aisle abstraction blocking model experiences less blocking delay due to the slower walk speed.

5.3. A limited number of picks and more pickers

The picker blocking estimates from Section 4 assume a potentially unlimited number of picks at a particular pick face and two pickers in the OPS. In this section we relax the two assumptions

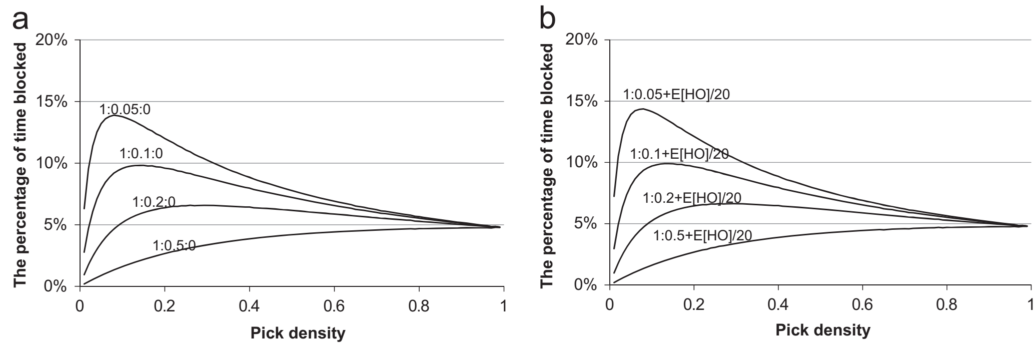

Fig. 14. Percentage of time blocked (two-picker, 20 pick faces) with infinite backward walk speed: (a) bucket brigade OPS; and (b) circular-aisle OPS with Corollary 1.

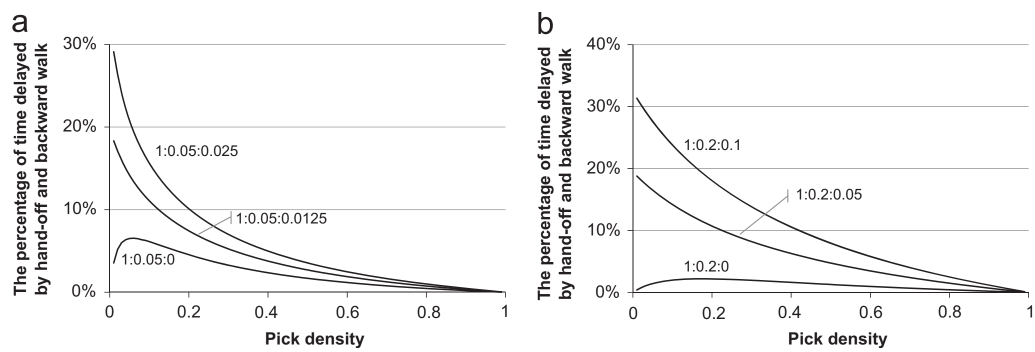

Fig. 15. Percentage of time blocked for the two-picker case with finite forward and backward walk speed: (a) forward walk speed = 0.05; and (b) forward walk speed = 0.2.

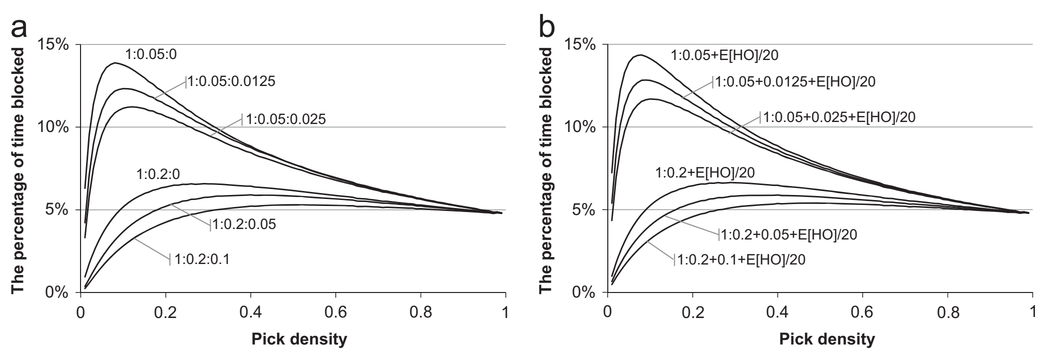

Fig. 16. Percentage of time blocked (two-picker, 20 pick faces) with finite backward walk speed: (a) bucket brigade OPS; and (b) circular-aisle abstraction OPS with Corollary 1′ compensation.

via simulation modeling to investigate the performance of our model under violations of these assumptions. The probability of v picks at a particular pick face is p^v. When p is large, the number of picks at a particular pick face can be large. We present simulation results with five pickers in the OPS limiting the maximum number of picks at a pick face (m)=2, 5, 10, 20, 50, and 100. We find the average error gap is 2.07%, with a maximum gap of 30.17% and a minimum gap of 0.44% between the simulation results and our analytical model. Therefore, in cases of a limited number of picks and more pickers, the result in Corollary 1′ holds, *i.e.*, the augmented circular-aisle abstraction model experiences a similar amount of productivity loss due to blocking delay. We note that even though larger batch sizes improve operational throughput, each batch requires a longer cycle time to reach completion, *i.e.*, order picking slows down. Depending on the urgency of the lead time, warehouse managers will need to set the size of batches appropriately (Fig. 17).

5.4. Batch picking with different capability

In general, bucket brigade OPSs are applied to dynamic order picking situations having a relatively small number of orders available in an order picking time window. We use an order picking profile based on Bartholdi and Eisenstein (1996a) and Koo (2009) to investigate three scenarios (Table 1). The base case is an order picking operation with 100 pick faces and five pickers. A picker performs with pick:forward walk:backward walk times in the ratio 1.0:0.1:0.05. The *standard* scenario assigns five identically-skilled pickers and uses the walk speed and picking capability configurations defined in Table 1. The *slowest-to-fastest ordering* scenario differentiates picking capabilities across pickers; the time per pick for the five pickers is 1.2, 1.1, 1.0, 0.9, and 0.8, where an average picker still performs one pick per unit time. In the *fastest-to-slowest ordering* scenario, the time per pick for the five pickers is 0.5, 0.75, 1.0, 1.25, and 1.5. We evaluate the three

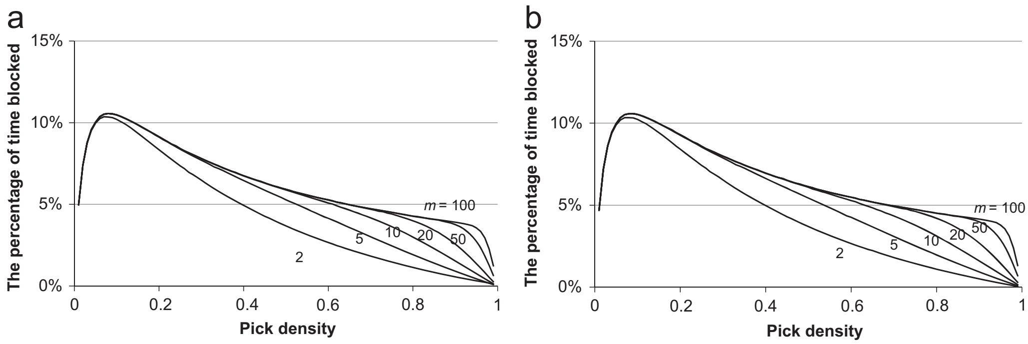

Fig. 17. Percentage of time blocked (five-picker, 100 pick faces) with finite backward walk speed: (a) bucket brigade OPS; and (b) circular-aisle OPS with Corollary 1′ compensation.

Table 1
Summary of experimental picking environments.

Configuration	Values
Scenarios	Standard (STD), slowest-to-fastest ordering (STF), fastest-to-slowest ordering (FTS)
Mean of order sizes	10, 20, 30, 40, 50, and 100
Number of items per order	Uniform distribution [min, max]=[mean/2, mean∗3/2]
Pick time	Triangular distribution [min, mode, max]=[0.5, 1.0, 1.5]
Forward walk time	0.025, 0.05, 0.1, 0.25, 0.5
Backward walk time	0.00125, 0.025, 0.05, 0.125, 0.25
Performance measure	Pick time percentage (%) and Time blocked percentage (%)
Runs per instance	20 runs with 2000 batches

scenarios by varying the average batch sizes among 10, 20, 30, 40, 50, and 100 items. We randomly select the size of each batch based on a uniform distribution [min, max]=[mean/2, mean∗3/2]. To reflect the randomness of pick times, we draw the value from a triangular distribution of [min, mode, max]=[0.5, 1.0, 1.5]. The scenarios assume deterministic forward and backward walk times. As a performance measure, we compare Pick time (%) – the percentage of time spent picking compared to overall operations – to Time blocked (%) – the productivity loss by picker blocking. The five operations are picking, forward walking, backward walking, hand-off delay, and blocking delay.

Tables 2 and 3 illustrate the time blocked percentage and the pick time percentage over the batch size variation when pt:ft:bt=1:0.1:0.05 and the variations of walk time when the batch size is 20. In the bucket brigade OPS, STF experiences less picker blocking than FTS. Since the hand-off difference between STF and FTS is not significant, STF always dominates FTS as measured by throughput, *i.e.*, the percentage of pick time. Recall that picker blocking in a bucket brigade OPS is due to the no-passing protocol and occurs between two neighboring pickers. As the neighboring pickers are assigned in a slow-to-fast order, the downstream picker always completes his/her work more quickly if the workloads of both pickers are evenly distributed. We conclude that bucket brigade OPS has an operational advantage over the variation of picker skills compared to circular-aisle abstraction OPSs. However, it requires an appropriate assignment of pickers.

The bucket brigade operational strategy balances workload between pickers (Bartholdi and Eisenstein, 1996b). The self-balancing and self-organizing characteristics result in stable zone assignment per picker (Bartholdi and Eisenstein, 1996a). Fig. 18 illustrates the variation of zone size per picker under different scenarios. We find that when workers have similar capabilities, the zone sizes are similar, and that zones of the most upstream and most downstream pickers are only slightly larger than the zones of the other pickers. The most upstream picker does not need to wait to take over a new batch from a loading station, *i.e.*, the most

Table 2
Comparison results over batch-size variations with pt:ft:bt=1:0.1:0.05.

Batch size	Scenarios							
	Bucket brigade OPS						Circular-aisle OPS with Corollary 1′ compensation	
	Pick time %			Time blocked %			Pick time %	Time blocked %
	STD	STF	FTS	STD	STF	FTS	STD	STD
10	36.70	36.02	33.03	5.81	2.87	14.56	36.82	5.50
20	51.11	50.99	44.06	8.27	3.73	20.30	51.21	8.10
30	58.97	59.69	49.64	9.56	4.12	23.32	59.04	9.44
40	64.02	65.44	53.18	10.19	4.30	24.94	64.06	10.14
50	67.46	69.62	55.32	10.73	4.36	26.43	67.49	10.69
100	75.60	80.12	60.22	12.02	4.48	29.75	75.56	12.07

Table 3
Comparison results over walk time variations when the batch size is 20.

Walk time ratio	Scenarios							
	Bucket brigade OPS						Circular-aisle OPS with Corollary 1′ compensation	
	Pick time %			Time blocked %			Pick time %	Time blocked %
	STD	STF	FTS	STD	STF	FTS	STD	STD
0.025:0.0125	68.48	70.56	56.98	14.20	7.69	28.90	68.76	13.84
0.05:0.025	61.65	62.59	51.95	11.72	5.90	25.42	61.86	11.43
0.1:0.05	51.11	50.99	44.06	8.27	3.73	20.30	51.21	8.10
0.2:0.1	37.63	37.02	33.60	4.70	1.78	14.18	37.68	4.59
0.5:0.25	20.56	20.11	19.34	1.66	0.41	7.64	20.59	1.51

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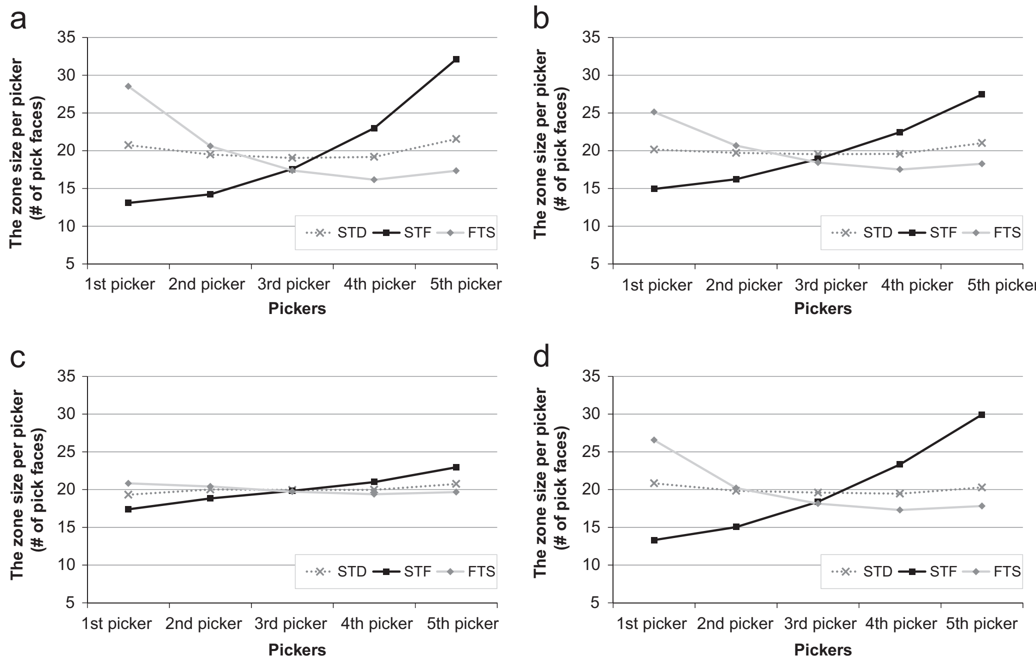

Fig. 18. Zone size per pickers (five-picker, 100 pick faces) with: (a) pt:ft:bt=1:0.025:0125 and batch size=20; (b) pt:ft:bt=1:0.1:0.05 and batch size=20; (c) pt:ft:bt=1:0.5:0.25 and batch size=20; and (d) pt:ft:bt=1:0.1:0.05 and batch size=100.

upstream picker extends the operational zone and has time to pick more items. Similarly, the most downstream picker has no downstream picker and is free from picker blocking. Thus, the most downstream picker takes advantage of extending the operational zone and has time to pick more items. The comparison of STF and FTS shows that skilled pickers under the STF scenario, *i.e*., pickers who can retrieve items more quickly, have larger zones than skilled pickers under the FTS scenario.

There are additional observations in the STF scenario. As walk speed becomes slower from Fig. 18(a), (b), to (c), the variations in zone size become small because a larger portion of time is used for walking. The increase in the overall workload also increases the variations in zone size, and is related to picker blocking. Picker blocking increases with the batch size and the variations in zone size are larger. Thus, the most downstream picker has more opportunities for picks. In the STD scenario, the faster walk speed and large batch size assigns a larger zone to the most downstream picker.

5.5. Insights

The amount of picker blocking increases when moving from the model with the assumption of no hand-off delay and the infinite backward walk speed compared to the model with the assumption of hand-off delay and finite backward walk speed. The release of the $i+PK$th batch, where PK represents the number of pickers, requires a period of time after the completion of the ith batch due to the hand-off delays and the backward walk times. Thus, the distance between $i+PK$th and $i+PK-1$st lengthens and picker blocking decreases. Some studies (Bartholdi and Eisenstein, 1996a, 1996b; Bartholdi et al., 2001) have assumed infinite walk speed and instantaneous hand-offs, but these assumptions lead to the overestimation of picker blocking. In fact, picker blocking lessens because more detailed models of bucket brigade order picking systems include finite forward and backward walk spends and account for hand-off delay. Our results support the previous finding that management can use bucket brigade OPSs with pickers sequenced from slowest-to-fastest to improve performance even when accounting for picker blocking explicitly.

6. Conclusions

This paper has made two important contributions to understanding bucket brigade OPS operations. First, an analytical model of picker blocking was constructed based on the interactions between two pickers under various walk speed assumptions. Second, based on the analytical model and additional simulation studies, the impacts by hand-off delay and backward walk speed were investigated. The modeling of both hand-off delays and finite backward walk speeds resulted in lower estimates of picker blocking delay.

The analytical models identified that a bucket brigade system experiences picker blocking when there is a difference of workload per pick face. Batch picking can reduce picker blocking, because it decreases the variance of workload per pick face. However, modeling pickers with varying levels of ability can also lead to relative differences in workload per pick face. Our study confirms previous results that when skill differences among pickers exist, management can sequence pickers from slowest-to-fastest to improve OPS performance. In addition, bucket brigade OPSs cause pickers to stay in a limited area and learn their operation environments.

Our assumptions regarding hand-off policies which induce hand-off delays maintains constant WIP in the system. Koo (2009) has suggested relaxing the hand-off policy and allowing more WIP in the system. Applying Little's law that states throughput is proportional to the ratio of WIP/cycle time to bucket brigade order picking, we recognize if WIP is constant, throughput inevitably drops as the cycle time increases which can be caused by picker blocking. Therefore, warehouse managers wanting to increase throughput should consider allowing WIP to increase as suggested by Koo (2009) or decrease the batch size to reduce the cycle time.

Our work suggests two streams of future research: 1) developing an integrated throughput model considering hand-off delays; and 2) identifying mitigation methods for less picker blocking. The hand-off delay in general is not an issue of management when the picking time is not significant. However, it becomes an issue as the pick time increases relative to the walk

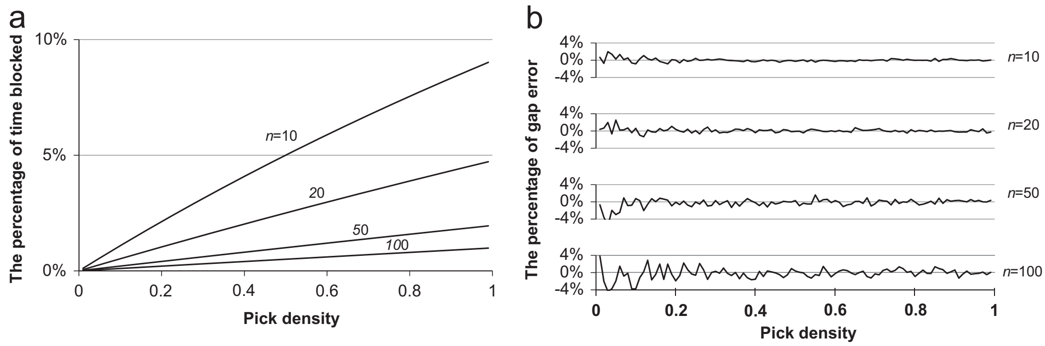

Fig. A1. (a) Percentage of time blocked in two-picker case with unit forward and instantaneous backward walk times (analytical results), and (b) percentage of gap error between simulation results and analytical results.

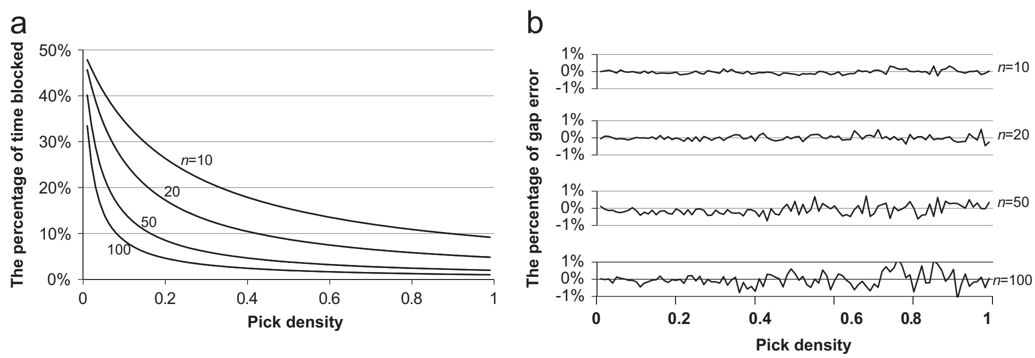

Fig. A2. (a) Percentage of time blocked for the two-picker case with infinite forward and backward walk speed (analytical results), and (b) percentage of gap error between simulation results and analytical results.

time. In bucket brigade order picking, picker blocking can be mitigated by allowing additional WIP (Koo, 2009). However, more WIP could slow operational performance (Cachon and Terwiesch, 2013). Thus, reducing picker blocking while holding WIP as low as possible is desirable.

Acknowledgments

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Appendix A. Comparison between analytical models and simulation results

We validate the analytical model in Eq. (3) *via* a comparison with the simulation models. A simulation comparison result for four different layouts ($n=10$, 20, 50, and 100) appears in Fig. A1. For each simulation, we specify the pick density on the range of 0.01–0.99 with the interval=0.01 with 20 runs per instance. Picker blocking becomes less significant when walk speed is very slow. The relative error gap between the analytical models and the four simulation results shows, on average, 0.52%, with a minimum gap of 0.00% and a maximum gap of 5.6%. Picker blocking may increase as the workload increases.

In addition, we can validate the analytical model in Eq. (5) *via* a comparison with the simulation models (Fig. A2). The error gap between the analytical models and the four simulation results is, on average, 0.19%, with a minimum gap of 0.00% and a maximum gap of 1.22%. If our two pickers instantaneously move forward and backward, picker blocking decreases as they spend more time picking.

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