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Estimating the shadow prices of SO_2 and NO_x for U.S. coal power plants: A convex nonparametric least squares approach ☆

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ABSTRACT

Weak disposability between outputs and pollutants, defined as a simultaneous proportional reduction of both outputs and pollutants, assumes that pollutants are byproducts of the output generation process and that a firm can "freely dispose" of both by scaling down production levels, leaving some inputs idle. Based on the production axioms of monotonicity, convexity and weak disposability, we formulate a convex nonparametric least squares (CNLS) quadratic optimization problem to estimate a frontier production function assuming either a deterministic disturbance term consisting only of inefficiency, or a composite disturbance term composed of both inefficiency and noise. The suggested methodology extends the stochastic semi-nonparametric envelopment of data (StoNED) described in Kuosmanen and Kortelainen (2011). Applying the method to estimate the shadow prices of SO_2 and NO_x generated by U.S. coal power plants, we conclude that the weak disposability StoNED method provides more consistent estimates of market prices.

1. Introduction

Coal power plants generate 47–56% of the electricity consumed in the U.S since 1989 (EIA, 2010). However, burning coal produces several byproduct pollutants, notably sulfur oxide (SO_2) and nitrogen oxide (NO_x), the major cause of acid rain. To address this problem, the Clean Air Act Amendments of 1990 (CAAA) set goals to reduce annual SO_2 emissions by 10 million tons and NO_x by 2 million tons from 1980 levels via a two-phase tightening of the restrictions placed primarily on coal plants (EPA, 2007). Phase I (1995–1999) regulated 445 boiler units at mostly coal plants and Phase II (2000–present) regulated over 2000 boiler units with a capacity greater than 25 MW at all fossil fuel plants. In 2011, the U.S. Environmental Protection Agency (EPA) released new environmental regulations requiring coal power plants to lower emissions of 84 toxic chemical levels within four years (EPA, 2011a).

An analysis of the effect of these regulations is helpful in understanding the impacts in terms of reductions in pollution and the associated costs for continued reductions. For this purpose we estimate a frontier production function, as first proposed by Farrell (1957). Data Envelopment Analysis (DEA), a technique named and popularized by Charnes et al. (1978), is extensively used to characterize firms' inputs usage to produce maximum level of outputs as well as to measure firms' technical efficiency. However, the original DEA model constructed a production frontier without modeling undesirable outputs such as pollutants. Consequently, Färe et al. (1986) extended DEA by applying Shephard's (1970) concept of weak disposability between desirable outputs and pollutants to estimate a production frontier and evaluate the impact of environmental regulations on technical efficiency. Today, the DEA weak disposability production frontier is applied to measure the firms' environmental performance. Färe et al. (1989) introduced a hyperbolic orientation to measure efficiency relative to DEA weak disposability frontier and applied the method to measure U.S. pulp and paper mills' technical efficiency and output losses due to environmental regulations. Yaisawarng and Klein (1994) measured productivity change of U.S. coal power plants by computing Malmquist input-based productivity assuming a DEA weak disposability frontier. Tyteca (1997) measured environment performance indicators of U.S. fossil fuel power plants based on a DEA weak disposability frontier. Pasurka (2006) calculated changes in SO_2 and NO_x associated with technical change, technical efficiency change and changes in input and output levels of U.S. coal power plants using an output distance function relative to a DEA weak disposability frontier. Mekaroonreung and Johnson (2010) used DEA and compared three approaches (hyperbolic efficiency measure; directional output distance function; linear transformation of pollutants)

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to estimate the technical efficiency of the U.S. oil refineries. See Zhou et al. (2008) for a summary of other DEA weak disposability applications in energy and environmental studies.

Recently, Sueyoshi and Goto (2011) proposed the concept of natural and managerial disposability and applied the concepts to a DEA frontier. A non-radial efficiency measure compared environmental performances and computed the returns to scale and damages to scale of national oil companies in several countries and international oil companies. This paper will focus on the more standard weak disposability assumption as the frontier for undesirable outputs implied by managerial disposability violates free disposability of inputs.

The implementation of the weak disposability assumption relative to a variable returns to scale (VRS) frontier has been subject to considerable debate. For instance, Färe and Grosskopf (2003) proposed a new model to construct a VRS weakly disposable production possibility set by introducing a single abatement factor across all firms whereas Kuosmanen (2005) used a non-uniform abatement factor across firms. In demonstrating that a production possibility set constructed by a single abatement factor model does not satisfy convexity, Kuosmanen and Podinovski (2008), proved that using non-uniform abatement factors allows the estimation of a VRS weakly disposable production possibility set that satisfies standard production axioms and the minimum extrapolation principle.

Many previous studies have estimated the shadow prices of undesirable outputs using distance functions. A ratio of the derivative of the distance function with respect to desirable output and the derivative of the distance function with respect to undesirable output characterizes the relative shadow price of the undesirable output, and parametric or nonparametric approaches can be used to estimate the distance function. The parametric approach is more widely used, because functions are everywhere differentiable. Färe et al. (1993) used an output distance function with the translog functional form to estimate a shadow price of four undesirable pollutants for 1976 data describing pulp and paper mills in Michigan and Wisconsin. Coggins and Swinton (1996) took the same approach to estimate the shadow price of SO_2 for Wisconsin coal plants in 1990–1992. Färe et al. (2005) used a quadratic directional output distance function to estimate both technical efficiency and a shadow price of SO_2 for the U.S. electric utilities in 1993 and 1997.

Despite its common usage, the parametric approach can be biased if the functional form is misspecified. Alternatively, a nonparametric approach, specifically DEA, can estimate a production frontier and the shadow prices of pollutants. Boyd et al. (1996) used a DEA production function to estimate the shadow price of SO_2 for coal plants. Lee et al. (2002) used DEA when accounting for technical inefficiency to derive the shadow prices of SO_2, NO_x and total suspend particulates (TSP) for Korean coal- and oil-burning plants in 1990–1995. Researchers also acknowledge some major limitations of the alternative approach: greater sensitivity to outliers, and the use of only a few observations to construct the production frontier. Moreover, DEA as a deterministic method does not incorporate statistical noise, and thus the observations of the production units must be observed without error and the production model specified without omitting any inputs or outputs.

Such drawbacks motivated the development of other nonparametric methods such as Convex Nonparametric Least Squares (CNLS), Kuosmanen (2006, 2008), which uses all available data to estimate a piecewise linear production function satisfying production axioms such as continuity, monotonicity and concavity. Kuosmanen and Johnson (2010) have shown that DEA is a special case of CNLS with sign constraints on error terms. To decompose statistical noise and inefficiency for cross-sectional data in a semi-parametric fashion, Kuosmanen and Kortelainen (2011) have proposed a two-stage method called Stochastic Non-parametric Envelopment of Data (StoNED).[1] It applies CNLS in the first stage to estimate an average production function and estimates the conditional expectation of inefficiency based on the CNLS residuals in the second stage.

The advantages of CNLS and StoNED over DEA motivated us to apply them to estimate a weak disposability production frontier. While DEA with weak disposability is well studied, to the best of our knowledge we are unaware of research that incorporates weak disposability with CNLS and StoNED. We describe our proposed model and apply it to measure the technical efficiency and to jointly estimate the shadow prices of SO_2 and NO_x for 196 U.S. coal power plants during Phase II of CAAA. To our knowledge there are no studies on the productive performance and shadow prices of SO_2 and NO_x using the U.S. coal power plants during Phase II of CAAA. The paper is organized as follows: the next section describes a nonparametric method of estimating a production function under weak disposability and the associated technical efficiency and shadow prices of SO_2 and NO_x. Section 3 describes the data set of 336 boilers of the U.S. bituminous coal power plants in operation from 2000 to 2008. Section 4 presents the analysis and discusses the results and Section 5 summarizes the conclusions.

2. Model

2.1. A production possibility set assuming weak disposability

For each firm $i=1..., n$ let $x \in R_+^M$ be a vector of inputs, $y \in R_+^S$ be a vector of good outputs and $b \in R_+^J$ be a vector of bad outputs. The production possibility set is defined as $T = \{(x,y,b): x \text{ can produce } (y,b)\}$. The assumptions defining the production possibility set are:

1. T is convex
2. There are variable returns to scale

Originally proposed by Shephard (1970), the following axioms regarding production are restated when undesirable outputs are also produced:

3. Free disposability of inputs
 If $(x,y,b) \in T$ and $x' \geq x$, then $\left(x',y,b\right) \in T$.
4. Free disposability of outputs
 If $(x,y,b) \in T$ and $y' \leq y$, then $\left(x,y',b\right) \in T$.
5. Weak disposability between outputs and pollutants
 If $(x,y,b) \in T$ and $0 \leq \varphi \leq 1$, then $(x, \varphi y, \varphi b) \in T$.

Based on the production possibility axioms stated above, the variable returns to scale weakly disposable production possibility set T can be written as:

$$T = \left\{ (x,y,b) \in R_+^{M+S+J} \middle| x \geq \sum_{i=1}^{n} (\lambda_i + \mu_i) x_i; y \leq \sum_{i=1}^{n} \lambda_i y_i; \right. \\ \left. b \geq \sum_{i=1}^{n} \lambda_i b_i; \sum_{i=1}^{n} (\lambda_i + \mu_i) = 1, \qquad \lambda_i, \mu_i \geq 0 \right\} \tag{1}$$

where $\lambda_i s$ allows the convex combination of observed firms and $\mu_i s$ allows firms to scale down both outputs and pollutants while maintaining the same level of inputs.

Formulation (1) differs from the Kuosmanen (2005) formulation in that the inequality sign in the pollutant constraints implies a negative shadow price on additional pollution and satisfies the economic intuition that pollutants incur costs to firms.

[1] See Johnson and Kuosmanen (2011) or http://www.nomepre.net/stoned/ for further discussion of this naming.

Using the weak disposable production possibility T in (1), the variable returns to scale output-oriented weak disposability DEA estimator can be written as:

$$
\begin{aligned}
&\max_{\theta_o,\lambda,\mu} \theta_o \\
&\text{s.t.} \sum_{i=1}^{n} \lambda_i y_{is} \geq \theta_o y_{os} && \forall s = 1,\ldots,S \\
&\sum_{i=1}^{n} \lambda_i b_{ij} \leq b_{oj} && \forall j = 1,\ldots,J \\
&\sum_{i=1}^{n} (\lambda_i + \mu_i) x_{im} \leq x_{om} && \forall m = 1,\ldots,M \\
&\sum_{i=1}^{n} (\lambda_i + \mu_i) = 1 \\
&\lambda_i, \mu_i \geq 0 && \forall i = 1,\ldots,n.
\end{aligned}
\tag{2}
$$

where y_{os}, b_{os}, x_{om}, and θ_o are outputs, bad outputs, inputs and the technical efficiency for specific firm o.

The DEA problem (2) constructs the weak disposability production frontier and estimates technical efficiency as the radial expansion of outputs.

2.2. Production frontier estimation

Consider a single output production function with a multiplicative disturbance term

$$y_i = f(x_i, b_i) \exp(\epsilon_i) \forall i = 1,\ldots,n \tag{3}$$

where $f(x_i, b_i)$ is the production function satisfying continuity, monotonicity, concavity and weak disposability and ϵ_i is the disturbance term. Note that the production function in (3) treats pollutants as independent variables following Cropper and Oates (1992), who defined this treatment as the standard approach to including pollutants within the environmental economics literature. Treating pollutants as independent variables has been used in several papers such as Pittman (1981) and Considine and Larson (2006).

Our motivations to employ a multiplicative disturbance model are twofold. First, as suggested in Kuosmanen and Kortelainen (2011), the multiplicative model allows the direct imposition of the assumptions of Constant Returns to Scale (CRS), Non-Increasing Returns to Scale (NIRS) or Non-decreasing Returns to Scale (NDRS). Specifically, CRS, NIRS and NDRS do not hold after an additive shift of the estimated frontier production function. Since the assumption of weak disposability between an output and pollutants requires the origin to be part of the convex production possibility set similar to the NIRS, the multiplicative disturbance term model is appealing. Second, the multiplicative disturbance term model helps to control for heteroskedasticity resulting from increased variability in output levels for production units operating at larger scale sizes.

Applying the log transformation to (3) gives:

$$\epsilon_i = \ln(y_i) - \ln(f(x_i, b_i)). \tag{4}$$

To estimate the weak disposability production frontier, we apply the CNLS technique to minimize the sum of the above multiplicative disturbances squared and assume the composition of ϵ_i to be

1. Deterministic (all deviations are attributed to inefficiency) or
2. Composite (mixture of inefficiency and random noise) or
3. Random (all deviations are random noise).

The results of estimating (4) by minimizing the sum of squared deviations *is* the production function under the assumption of random disturbances. If the disturbances are assumed to be deterministic, we could apply a one-stage method by solving the CNLS problem with sign constraints on the disturbances. The results of applying a one-stage method define an estimated production frontier and technical efficiencies. If the disturbance terms are assumed to be a mixture, we could apply the two-stage StoNED method, solving the CNLS problem with no sign constraint on the disturbances and then decomposing the CNLS residuals into statistical noises and technical efficiencies using Jondrow et al. (1982) see also Kuosmanen and Kortelainen (2011). The estimated averaged CNLS production function is shifted by the average technical efficiency level to obtain a production frontier. Below, we elaborate on these deterministic, composite and random disturbance term assumptions.

2.2.1. Deterministic disturbance term

We assume that there is no statistical noise in the data; thus, any deviations from the estimated frontier are due to technical efficiency. Specifically:

$$\epsilon_i = -u_i \forall i = 1,\ldots,n \tag{5}$$

where $u_i \geq 0$ is the firm-specific technical inefficiency.

While noting that the CNLS objective function is to minimize the sum of square disturbances, when all of the disturbances are less than or equal to zero in the deterministic case, we can replace the sum of square disturbances by the sum of disturbances, see Kuosmanen and Johnson (2010). The CNLS problem is then formulated as:

$$
\begin{aligned}
&\min_{\alpha,w,c,\epsilon} -\sum_{i=1}^{n} \epsilon_i \\
&\text{s.t.} \quad \epsilon_i = \ln(y_i) - \ln\left(\alpha_i + w_i' x_i + c_i' b_i\right) && \forall i = 1,\ldots,n \\
&\alpha_i + w_i' x_i + c_i' b_i \leq \alpha_h + w_h' x_i + c_h' b_i && \forall i,h = 1,\ldots,n \\
&\alpha_i + w_i' x_h \geq 0 && \forall i,h = 1,\ldots,n \\
&w_i, c_i \geq 0, \epsilon_i \leq 0 && \forall i = 1,\ldots,n.
\end{aligned}
\tag{6}
$$

The objective function maximizes the sum of the disturbance terms. Intuitively, the CNLS problem (6) estimates a production frontier that makes all firms look as efficient as possible using the minimum extrapolation principle of Banker et al. (1984), also referred to as the benefit of the doubt principle by Moesen and Cherchye (1998). The first equality constraints define the disturbance term. The second inequality constraints comprise a system of Afriat inequalities, Afriat (1972), imposing the underlying production function to be continuous and concave. The third inequality constraints impose the weak disposability between desirable and undesirable outputs. The last constraints enforce monotonicity of both inputs and the costs associated with additional undesirable outputs.

Solving the CNLS problem (6) obtains the production frontier. Technical efficiency is obtained from the estimated CNLS residual, $\hat{\epsilon}_i$, $\forall i$:

$$TE_i = \exp(\hat{\epsilon}_i) \forall i = 1,\ldots,n. \tag{7}$$

Proposition 1. In a single output case, the deterministic CNLS production function (6) is equivalent to the output-oriented weak disposability DEA production function (2).

Proof. See Appendix.

Proposition 2. The technical efficiency estimates from (7) equal the reciprocal of the technical efficiency estimates from DEA (2).

Proof. See Appendix.

DEA or CNLS with a deterministic disturbance as described above can be used to nonparametrically estimate a weak disposability production function; however, it is not appropriate if the production model is imperfectly specified or the data set contains noise. Again, CNLS is more advantageous because it can be extended to estimate weak disposability production functions including a model of statistical noise.

2.2.2. Composite disturbance term

Similar to Stochastic Frontier Analysis (SFA) by Aigner et al. (1977), we assume that there is statistical noise in the data; thus any disturbance terms can be written as:

$$\epsilon_i = v_i - u_i \forall i = 1, \ldots, n \tag{8}$$

where v_i is a random noise component.

As Kuosmanen and Kortelainen (2011) have pointed out, the composite disturbance term in (8) violates the Gauss–Markov properties that $E(\epsilon_i) = E(-u_i) = -\mu < 0$ where μ is the expected technical inefficiency. Therefore, we modify the composite disturbance term in (8). The multiplicative disturbance production model $y_i = f(x_i, b_i)\exp(\epsilon_i)$ is written as:

$$\ln(y_i) = [\ln(f(x_i,b_i)) - \mu] + [\epsilon_i + \mu] = \ln(g(x_i,b_i)) + \vartheta_i \forall i = 1, \ldots, n \tag{9}$$

where $\vartheta_i = \epsilon_i + \mu$, the modified composite disturbance term. Note that $E(\vartheta_i) = E(\epsilon_i + \mu) = 0$. The CNLS problem is formulated as:

$$\begin{aligned} &\min_{\alpha,w,c,\vartheta} \sum_{i=1}^{n} \vartheta_i^2 \\ &\text{s.t.} \quad \vartheta_i = \ln(y_i) - \ln\left(\alpha_i + w_i' x_i + c_i' b_i\right) && \forall i = 1, \ldots, n \\ &\qquad \alpha_i + w_i' x_i + c_i' b_i \le \alpha_h + w_h' x_i + c_h' b_i && \forall i, h = 1, \ldots, n \\ &\qquad \alpha_i + w_i x_h \ge 0 && \forall i, h = 1, \ldots, n \\ &\qquad w_i, c_i \ge 0 && \forall i = 1, \ldots, n. \end{aligned} \tag{10}$$

Because CNLS identifies a production function that minimizes the sum of squared disturbances among all production functions that are continuous, monotonic increasing, concave and satisfy the weak disposability assumptions, it is important to check the following condition for the objective function.

Proposition 3. The objective function in the CNLS problem (10) is a convex function if and only if $\frac{y_i}{\alpha_i + w_i' x_i + c_i' b_i} \ge \frac{1}{e} \forall i = 1, \ldots, n$.

Proof. See Appendix.

If the CNLS problem (10) has a convex objective function, then a local optimum to (10) is also a global optimum simplifying the optimization algorithms needed to find the global optimal solution to (10).

The second stage of CNLS separates the technical efficiency and statistical noise components using the estimated modified CNLS residuals $\hat{\vartheta}_i \forall i$ from (10). Assuming that technical efficiency is independent and identically distributed (i.i.d.) and has a half normal distribution and that the statistical noise is i.i.d. and normally distributed, $u_i \sim |N(0, \sigma_u^2)|$ and $v_i \sim N(0, \sigma_u^2)$, the method of moments can be applied (Aigner et al. (1977) as in Kuosmanen and Kortelainen (2011)). Specifically,

$$\hat{\sigma}_u = \sqrt[3]{\frac{\hat{M}_3}{\left(\frac{2}{\pi}\right)\left(1 - \frac{4}{\pi}\right)}} \text{ and } \hat{\sigma}_v = \sqrt{\hat{M}_2 - \left(\frac{\pi - 2}{\pi}\right)\hat{\sigma}_u^2} \tag{11}$$

where $\hat{M}_2 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\vartheta}_i - \hat{E}(\vartheta_i)\right)^2$ and $\hat{M}_3 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\vartheta}_i - \hat{E}(\vartheta_i)\right)^3$.

Unlike the deterministic disturbance case, after solving the CNLS problem (10), the average production function $g(x_i, b_i)$ is obtained instead of the production frontier. Next, the average production function is multiplied by the expected technical efficiency to estimate the production frontier. Specifically,

$$\begin{aligned} &\ln(\hat{g}(x_i,b_i)) = \left[\ln\left(\hat{f}(x_i,b_i)\right) - \hat{\mu}\right] = \ln\left(\hat{f}(x_i,b_i)\exp(-\hat{\mu})\right), \text{thus} \\ &\hat{f}(x_i,b_i) = \hat{g}(x_i,b_i)\exp(\hat{\mu}) \end{aligned} \tag{12}$$

where $\hat{\mu} = \hat{\sigma}_u\sqrt{\frac{2}{\pi}}$.

Given $\hat{\sigma}_u$ and $\hat{\sigma}_v$, the method introduced in Jondrow et al. (1982) can be used to estimate firm-specific inefficiency. Specifically,

$$\hat{E}(u_i|\hat{\epsilon}_i) = -\frac{\hat{\epsilon}_i \hat{\sigma}_u{}^2}{\hat{\sigma}_u{}^2 + \hat{\sigma}_v{}^2} + \frac{\hat{\sigma}_u{}^2 \hat{\sigma}_v{}^2}{\hat{\sigma}_u{}^2 + \hat{\sigma}_v{}^2}\left[\frac{\phi\left(\hat{\epsilon}_i/\hat{\sigma}_v{}^2\right)}{1 - \Phi\left(\hat{\epsilon}_i/\hat{\sigma}_v{}^2\right)}\right] \tag{13}$$

where $\hat{\epsilon}_i = \hat{\vartheta}_i - \hat{\mu}$, ϕ is the standard normal density function and Φ is the standard normal cumulative distribution.

2.2.3. Random disturbance term

Here, the CNLS estimator in (4) is used to obtain the residual directly. Assuming statistical noise is i.i.d. and normally distributed, $v_i \sim N(0, \sigma_v^2)$, the method of moments is applied in the second stage of CNLS to estimate σ_v. Specifically, $\hat{\sigma}_v = \hat{M}_2$ where $\hat{M}_2 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\vartheta}_i - \hat{E}(\vartheta_i)\right)^2$.

We note that composite disturbances and random disturbances differ in that the former is skewed by inefficiency. This skewness can be used to determine if the neoclassical production function (no inefficiency) or the frontier production function (with inefficiency) is the proper model. In Section 4, we use the test proposed by Kuosmanen and Fosgerau (2009) to select between a neoclassical or frontier production function based on the skewness of the residuals. The results provide evidence for the presence of inefficiency.

2.3. Estimating shadow prices of pollutants

Assuming profit-maximizing behavior for each firm, the profit maximization problem for a production process with outputs and pollutants is

$$\begin{aligned} &\pi\left(p_y, p_b, p_x\right) = \max_{y,b,x} p_y{}'y - p_b{}'b - p_x{}'x \\ &\text{s.t.} \quad F(x,b,y) = 0 \end{aligned} \tag{14}$$

where $p_y = (p_{y_1}, \ldots, p_{y_S})$, $p_b = (p_{b_1}, \ldots, p_{b_J})$ and $p_x = (p_{x_1}, \ldots, p_{x_M})$ represent the price vectors of outputs, pollutants and inputs, respectively. $F(x, b, y)$ is the transformation function corresponding to a multi-output production function. Since we are interested in the shadow prices of pollutants, we impose the constraint $F(x, b, y) = 0$ so that only the frontier of the production possibility set is considered. Problem (15) applies the method of Lagrangian multipliers to (14)

$$\max_{y,b,x} p_y{}'y - p_b{}'b - p_x{}'x + \zeta F(x,b,y) \tag{15}$$

where ζ is a Lagrangian multiplier of the constraint.

The first-order conditions (FOCs) of the problem (15) are:

$$\begin{aligned} &p_{y_s} + \zeta\frac{\partial F(x,b,y)}{\partial y_s} = 0 \\ &-p_{b_j} + \zeta\frac{\partial F(x,b,y)}{\partial b_j} = 0 \\ &-p_{b_m} + \zeta\frac{\partial F(x,b,y)}{\partial x_m} = 0 \\ &\qquad 0 = F(x,b,y). \end{aligned} \tag{16}$$

The shadow prices can be written as:

$$p_{b_j} = p_{y_s}\left(\frac{\partial F(x,b,y)}{\partial b_j} \Big/ \frac{\partial F(x,b,y)}{\partial y_s}\right). \tag{17}$$

In the case of a single output production function ($S=1$), the first equality of the FOC (16) can be written as $p_y - \zeta = 0$, that is, the Lagrangian multiplier is equal to the price of *the* output. Thus, if the price of *the* output is known, the shadow prices of each pollutant can be estimated using the second equality in (16). The relative shadow prices of pollutants for firm i are estimated as:

$$p_{b_{ij}} = p_{yi}\frac{\partial f(x_i,b_i)}{\partial b_{ij}} \tag{18}$$

where p_{yi} is the price of an output for firm i.

Assuming a deterministic disturbance term, solving the CNLS problem (6) estimates a weak disposability production frontier $\hat{f}(x_i,b_i)$ directly. We obtain the variable $\frac{\partial \hat{f}(x_i,b_i)}{\partial b_{ij}}$ for each firm from the estimated variable $\hat{c}_{ij} \in \hat{c}_i = (\hat{c}_{i1}, \hat{c}_{i2}, \ldots, \hat{c}_{iJ})$ in (6) directly.[2]

Under the composite disturbance term assumption, in the first stage we estimate the average weak disposability production function $\hat{g}(x_i,b_i)$ by solving the CNLS problem (10). In the second stage, we decompose the estimated composite CNLS residuals and calculate the estimated expected inefficiency components $\hat{\mu}$. We obtain the relative shadow prices of pollutants for each firm as $\frac{\partial \hat{g}(x_i,b_i)}{\partial b_{ij}}\exp(\hat{\mu}) = \hat{c}_{ij}\exp(\hat{\mu})$, where the variable $\hat{c}_{ij} \in \hat{c}_i = (\hat{c}_{i1}, \hat{c}_{i2}, \ldots, \hat{c}_{iJ})$ is obtained from solving (10).

3. Data set

Our balance panel boiler-level data characterizes 336 units of the U.S. bituminous coal-burning electricity plants in operation from 2000 to 2008. Bituminous coal plants are mostly located in the eastern states and these power plants produce about 50% of the total electricity generated from coal. This form of coal has very high sulfur content. All boilers in the sample are either wall or tangential fired boilers, sub-groups of pulverized coal-fired boilers, which are regulated by the Acid Rain Program.

The output is the annual amount of electricity generated (in Megawatt-hours, MWh). The pollutants are the annual amount of SO_2 (tons) and NO_x (tons). The two inputs are capital and heat. The heat input (mmBtu), calculated by multiplying the quantity of fuel with the fuel's heat content, is the measure of fuel utilization. Information on electricity generated, amount of pollutants and heat input quantities are reported by the EPA database (EPA, 2011b).

The boiler size (MW), the maximum rated output of a generator under specific conditions, is used as an instrumental variable for capital. The U.S. EPA's database reported the maximum heat input capacity (mmBtu/h), a unit's maximum designed hourly heat input rate observed in the past five years, for each boiler unit. We convert the maximum heat input capacity to estimate the boilers' sizes. The boilers' sizes in our sample range between 100 and 1426 MW.

[2] The CNLS production frontier is piecewise linear. Infrequently observations lie on a kink of the CNLS production frontier and do not have unique shadow prices. Using the method proposed by Kuosmanen and Kortelainen (2011), we use the minimum marginal product of pollutants to estimate shadow prices.

Electricity prices ($/MWh) of each utility are reported in EIA861 (EIA, U.S. Energy Information Administration, 2011b). Some of the utilities do not generate electricity; thus, we match our power plants in the sample to those utilities in which they have electricity production data and assume that electricity price in those utilities are the same as in power plants. Following Färe et al. (2005) approach, we assume that all generating boilers in the same power plants have the same electricity prices. We derived electricity prices for each boiler by the average price of electricity sales for customers and for resale of each corresponding utility. For some utilities without electric price information, we use the state average retail electricity price reported in EIA (2011a).

From the original 491 bituminous coal power plant boilers data we are able to collect, we construct the 9 years balance panel data set based on the input output information described above. We drop 97 boilers for which their size are less than 100 MW, 55 boilers for which they are not pulverized coal-fired boilers and 3 boilers for which there are missing data on electricity and pollutants, leaving 336 boilers units in the sample. There was no entry or exit of coal power plants observed in the data gathered over this time horizon. The summary statistics are presented in Table 1.

Table 1
Statistics for boiler units in the U.S. coal electricity plants (n = 336).

Year	Variable[a]	Mean	Std. dev.	Min.	Max.
2000	Electricity	2141	1640	257	8315
	SO_2	12.57	11.45	0.30	76.28
	NO_x	4.69	3.55	0.78	18.68
	Heat input	20,855	15,672	3201	79,135
	Price	52.26	14.51	17.26	113.80
2001	Electricity	2029	1618	236	10,378
	SO_2	11.64	10.79	0.25	63.57
	NO_x	4.31	3.34	0.52	20.89
	Heat input	19,741	15,306	2283	86,749
	Price	54.13	17.45	20.71	115.50
2002	Electricity	2062	1689	261	10,474
	SO_2	11.33	10.73	0.24	87.59
	NO_x	4.22	3.42	0.36	20.97
	Heat input	19,945	15,762	2737	88,046
	Price	50.44	17.06	21.44	111.60
2003	Electricity	2127	1721	242	10,210
	SO_2	12.02	12.07	0.26	83.56
	NO_x	3.99	3.19	0.69	20.17
	Heat input	20,412	15,854	2291	92,378
	Price	52.83	16.60	21.54	124.40
2004	Electricity	2097	1700	251	9940
	SO_2	11.67	11.38	0.22	75.75
	NO_x	3.57	2.80	0.40	15.17
	Heat input	20,027	15,674	2526	83,167
	Price	55.13	17.46	22.42	125.50
2005	Electricity	2166	1785	266	11,155
	SO_2	11.81	11.85	0.19	80.98
	NO_x	3.48	2.73	0.42	15.15
	Heat input	20,605	16,299	2803	92,853
	Price	59.65	18.67	24.71	139.50
2006	Electricity	2160	1749	200	10,363
	SO_2	11.25	11.40	0.20	71.92
	NO_x	3.42	2.74	0.38	16.59
	Heat input	20,401	15,939	2191	83,026
	Price	65.43	19.36	28.58	154.50
2007	Electricity	2187	1765	83	10,094
	SO_2	10.34	11.40	0.13	92.63
	NO_x	3.30	2.75	0.31	14.78
	Heat input	20,789	16,388	886	95,973
	Price	67.53	20.85	23.57	152.20
2008	Electricity	2180	1763	83	10,094
	SO_2	10.31	11.41	0.13	92.63
	NO_x	3.30	2.75	0.31	14.78
	Heat input	20,740	16,388	886	95,973
	Price	73.98	22.04	36.06	165.70
	Boiler size	336	240	100	1426

[a] Unit of electricity, SO_2, NO_x, heat input, electricity price and boiler size are $10^3 \times$ MWh, $10^3 \times$ ton, $10^3 \times$ ton, $10^3 \times$ mmBTU, $/MWh and MW, respectively.

Table 2
Results of the skewness and kurtosis tests.

Year	Test statistics		P-values	
	$\sqrt{b_1}$	b_2	$\sqrt{b_1}$	b_2
2000	−0.084	3.987	0.263	0.003
2001	−0.271	4.022	0.020	0.003
2002	−0.382	4.069	0.002	0.002
2003	−0.546	4.004	0.000	0.003
2004	−0.496	4.738	0.000	0.000
2005	−0.168	3.334	0.102	0.095
2006	−0.618	4.785	0.000	0.000
2007	−0.560	5.334	0.000	0.000
2008	−0.488	5.318	0.000	0.000

4. Empirical results

To test whether the assumption of the frontier production function is more appropriate than the neoclassical production function, we apply the skewness and kurtosis tests proposed in Kuosmanen and Fosgerau (2009). The null hypothesis H_0 is: disturbances that are normally distributed are tested against an alternative hypothesis H_1: disturbances are negative skewed.[3] Table 2 reports the $\sqrt{b_1}$ and b_2 test statistics and the relevant p-values of the normality tests. As expected, the $\sqrt{b_1}$ statistics are negatively signed. At the 5% significance level, normality is rejected in favor of skewness in 2001–2004 and 2006–2008, which supports the frontier model, and cannot be rejected in 2000 and 2005, which supports the neoclassical assumption. Thus, in these two years we use the neoclassical production model in which the disturbances contain only noise.

Table 3
Statistics of the estimated shadow prices of SO_2 and NO_x ($/ton) and technical inefficiency for the deterministic disturbance term case.

Variable	Mean[a]	Std. dev.[a]	Min	Max
2000				
$Price_{SO2}$	743	11,140	0	368,805
$Price_{NOx}$	7536	9053	0	130,271
TE	0.884	0.077	0.674	1.000
2001				
$Price_{SO2}$	1291	16,816	0	391,923
$Price_{NOx}$	9234	11,713	0	58,875
TE	0.879	0.077	0.665	1.000
2002				
$Price_{SO2}$	507	1041	0	18,176
$Price_{NOx}$	4597	6425	0	52,715
TE	0.888	0.072	0.685	1.000
2003				
$Price_{SO2}$	691	2712	0	79,618
$Price_{NOx}$	6187	11,262	0	80,503
TE	0.886	0.073	0.637	1.000
2004				
$Price_{SO2}$	734	4296	0	125,749
$Price_{NOx}$	7799	11,272	0	75,287
TE	0.883	0.073	0.612	1.000
2005				
$Price_{SO2}$	1033	11,274	0	358,869
$Price_{NOx}$	11,679	13,692	0	75,962
TE	0.892	0.066	0.697	1.000
2006				
$Price_{SO2}$	954	9457	0	331,175
$Price_{NOx}$	9994	13,356	0	81,886
TE	0.902	0.063	0.700	1.000
2007				
$Price_{SO2}$	780	8074	0	229,204
$Price_{NOx}$	4044	7367	0	83,184
TE	0.875	0.075	0.684	1.000
2008				
$Price_{SO2}$	2020	20,215	0	458,105
$Price_{NOx}$	3671	5583	0	73,031
TE	0.869	0.077	0.618	1.000

[a] Weighted average by the amount of pollutants.

Table 4
Statistics of the estimated shadow prices of SO_2 and NO_x ($/ton) and technical inefficiency for the composite disturbance term case.

Variable	Mean[a]	Std. dev.[a]	Min	Max
2000				
$Price_{SO2}$	201	255	0	2573
$Price_{NOx}$	1354	1281	0	8035
TE	N/A	N/A	N/A	N/A
2001				
$Price_{SO2}$	293	388	0	3218
$Price_{NOx}$	848	1649	0	11,955
TE	0.943	0.036	0.818	1
2002				
$Price_{SO2}$	318	453	0	4899
$Price_{NOx}$	811	1542	0	16,542
TE	0.938	0.042	0.801	1
2003				
$Price_{SO2}$	230	418	0	3338
$Price_{NOx}$	691	2047	0	17,648
TE	0.927	0.049	0.743	1
2004				
$Price_{SO2}$	219	410	0	3215
$Price_{NOx}$	1211	3344	0	30,712
TE	0.934	0.044	0.740	1
2005				
$Price_{SO2}$	246	1467	0	37,648
$Price_{NOx}$	1352	3703	0	19,071
TE	N/A	N/A	N/A	N/A
2006				
$Price_{SO2}$	343	3097	0	108,436
$Price_{NOx}$	1301	3687	0	32,640
TE	0.935	0.043	0.757	1
2007				
$Price_{SO2}$	237	421	0	4907
$Price_{NOx}$	409	1456	0	16,775
TE	0.937	0.042	0.777	1
2008				
$Price_{SO2}$	239	666	0	11,572
$Price_{NOx}$	609	2082	0	28,770
TE	0.931	0.046	0.745	1

[a] Weighted average by the amount of pollutants.

Table 3 reports the estimated shadow prices of SO_2 and NO_x, technical inefficiencies and related statistics, assuming a deterministic disturbance. The estimated average prices of SO_2 over the 9-year time horizon, range between 509 and 2020 $/ton and the estimated average prices of NO_x are between 3671 and 11,679 $/ton. The estimated average technical inefficiencies range between 0.883 and 0.902.

Table 4 shows the estimated shadow prices of SO_2 and NO_x using StoNED and related statistics. The values in parentheses represent the statistics excluding zero shadow price firms. The estimated average shadow prices of SO_2 range between 201 and 343 $/ton and the estimated average shadow prices of NO_x between 409 and 1352 $/ton.

[3] The simulated distribution of the skewness test statistic, $\sqrt{b_1}$, and the kurtosis test statistic, b_2, are constructed by a simple Monte Carlo simulation using $M=10{,}000$ Pseudo-samples of $n=336$ observations from $N(0,1)$.

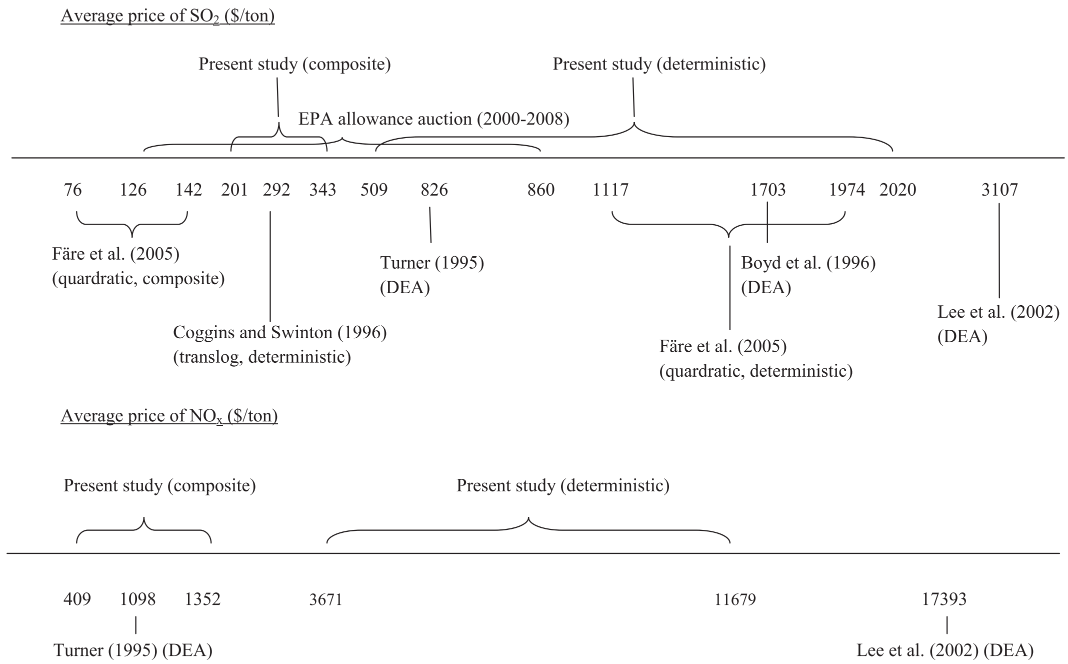

Fig. 1. Comparison of the estimated average shadow prices of SO_2 and NO_x.

The estimated average technical inefficiencies range between 0.927 and 0.943. For all data sets, the estimated second and third moments of the residual, $\hat{M}_2$ and $\hat{M}_3$, have the correct signs; thus, the expected inefficiency terms can be calculated and used to estimate the shadow prices of both pollutants. The convexity condition, $\frac{y_i}{\hat{\alpha}_i + \hat{w}_i{}' x_i + \hat{c}_i' b_i} \geq \frac{1}{e} \approx 0.368 \forall i$, in proposition 3 is satisfied for each year of data, which indicates that the objective function is globally convex. Therefore, a global optimal solution can always be found using standard nonlinear programming methods.

We find that applying the deterministic methods results in higher shadow price estimates than when assuming a composite disturbance term. Moreover, the estimated shadow prices in the deterministic case have a wider range. Weak disposability production frontiers constructed in the deterministic case are more sensitive to variation. If outliers are present in the data set, the estimated frontier tends to have larger steep regions, thus $\frac{\partial \hat{f}(x_i, b_i)}{\partial b_{ij}}$ is large and the estimated shadow prices are higher. In general, when only a few extreme observations are used to construct a frontier, the result is more variation in the estimated shadow price.

Fig. 1 shows the average shadow price estimates compared with previous studies; note that every study uses different data sets and estimation methods as summarized in Table 5. From Fig. 1, three conclusions can be drawn. First, average shadow price estimates for SO_2 from previous studies ranges between 76 to 3107 \$/ton, is consistent with the previous literature and EPA auction prices. For the composite disturbance model, our average shadow price estimates for SO_2, ranging from 201 to 343 \$/ton, contain the estimates of Coggins and Swinton (1996) and are close to Färe et al. (2005). More importantly, our estimates from the composite disturbance model are in the range of EPA's SO_2 allowance auction prices.

The results also confirm that the shadow price estimates from composite disturbance models are generally lower than those from deterministic models, and are likely better estimates of the prices from the EPA's allowance markets. Excluding Coggins and Swinton (1996),[4] the average shadow price for SO_2 from deterministic models (including DEA) are 509–3107 \$/ton compared to 76–343 \$/ton for the composite models. Table 6 shows that the SO_2 market prices are 130–1550 \$/ton and the allowance auction price is 126–860 \$/ton.[5] We conclude that the weak disposability StoNED method provides more consistent estimates of market prices compared to weak disposability DEA. Third, the shadow prices of NO_x are higher than the shadow prices of SO_2. Using a composite disturbance term, the NO_x average shadow prices of 409–1352 \$/ton are higher than the SO_2 shadow prices of 201–343 \$/ton. This conclusion coincides with the observed prices in the SO_2 and NO_x allowance markets.

Table 6 shows the comparison of average shadow price estimates in the present study to the pollutants' market prices. Our average SO_2 prices are slightly over-estimated between 2000 and 2003, within the range for 2004 and 2008 and under-estimated between 2005 and 2007, because our average SO_2 price estimates are relatively stable year to year while the SO_2 market prices starts to increase in 2003,

Table 5
Data set comparisons of the electricity price used to estimate shadow prices of the pollutants.

Study	Country	Year	Sample size	Price of electricity (\$/MWh)
Boyd et al. (1996)	U.S.	1989	62	50.00
Coggins and Swinton (1996)	U.S.	1990–1992	42	36.38–65.87
Färe et al. (2005)	U.S.	1993, 1997	209	10.39–100.42
Lee et al. (2002)	Korea	1990–1995	43	66.67
Present study	U.S.	2000–2008	336	17.26–165.70

[4] Compared to other studies, this paper considers a limited number of boilers in Wisconsin all facing similar state regulations. The boilers in this study tend to have similar production characteristics; thus, if the data was collected carefully, the assumption a deterministic disturbance is more appropriate than in other studies with more heterogeneity and noise.

[5] Allowance auction price is the price for which the allowance is sold to the highest bidder in the annual EPA auction until no allowances remain. Market price is the price for which the allowance is traded on the open market.

Table 6
Comparison between pollutants market prices[a] ($/ton) and the average shadow price estimates from the present study using composite disturbance case ($/ton).

Year	SO_2 prices		NO_x prices[b]	
	Market	Present study	Market	Present study
2000	130–155	201		1354
2001	135–210	293	600–1700[c]	848
2002	130–170	318		811
2003	150–220	230	2500–8000	691
2004	215–700	219	2100–3700	1211
2005	700–1550	246	2000–3500	1352
2006	430–740	343	900–2725	1301
2007	500–600	237	500–1000	409
2008	179–509	239	592–1400	609

[a] Information on SO_2 allowance market prices is reported in EPA Acid Rain Program 2000–2008 Progress Reports, information on NO_x allowance market prices is reported in EPA OTC NO_x Budget Trading Program 1999–2002 Progress Reports and EPA NO_x Budget Trading Program 2003–2008 Progress Reports. All reports are available at: http://www.epa.gov/airmarkt/progress/progress-reports.html.

[b] For 2003, 2004 and 2005, the range of NO_x market prices are approximated from graphs; for the other years, the range of NO_x market prices are explicitly stated in the EPA reports.

[c] For 2000, 2001 and 2002, EPA published three years of progress in a single OTC NO_x budget program report.

spikes during 2004–2005 and declines after 2005. Our average NO_x prices are within the range or close to NO_x market prices except in 2003, 2004 and 2005. During this time, our estimates are lower than the market price because the NO_x market price increases sharply. However, our NO_x price estimates have a similar trend of rising prices in 2004, 2005 and 2006 and dropping prices in 2007 and 2008. Figs. 2 and 3 illustrate the average shadow price estimates and the market prices.

Recall that the shadow prices of pollutants are estimates of the marginal abatement costs which should reflect the market prices for EPA's pollutant allowances. Our shadow price estimates are derived based solely on the plants' production data; however, several other factors can affect the market allowance price. By allowing plants to buy, sell and bank allowances, the allowance prices reflect the cost of compliance with future regulation. The sharp increase in SO_2 and NO_x prices resulting from EPA's Clean Air Interstate Rule (CAIR) which requires further SO_2 and NO_x reduction from coal boilers beginning in 2010, caused an increase in the expected pollutant control costs in the future and provided incentives for plants to buy allowances and bank them for future use. Thus, allowance prices rose due to increased demand for allowances. After 2005, emission levels fell due to the increased use of gas-fired boilers and pollution

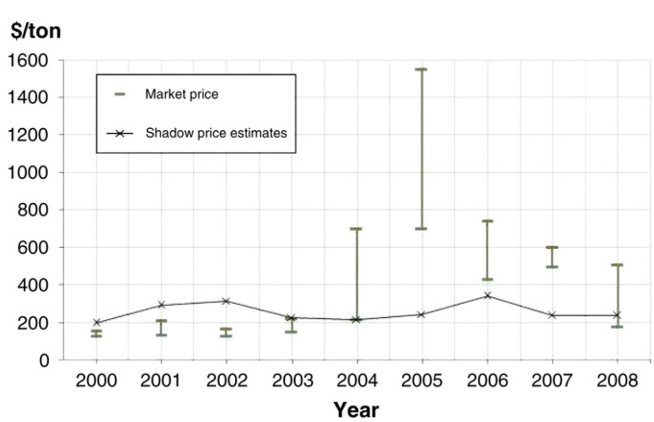

Fig. 2. SO_2 market prices and the average shadow price estimates ($/ton).

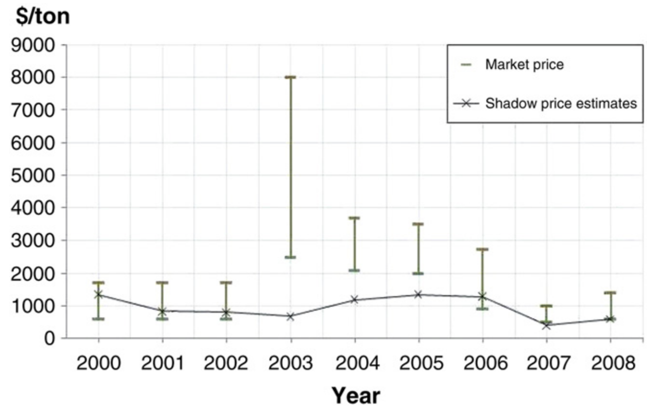

Fig. 3. NO_x market prices and the average shadow price estimates ($/ton).

control equipment. Thus, a sufficient supply of allowances in the market caused allowance market prices to fall.

5. Conclusions

This paper proposed a nonparametric methodology to estimate a production frontier when pollutants are a result of the production process. We assumed that the traditional production axioms such as continuity, monotonicity and concavity with weak disposability between an output and the pollutants characterized the shape of an underlying production frontier. To estimate the production frontier empirically, we extended the two-stage CNLS method to incorporate the weak disposability assumption. In deterministic cases assuming no noise in the data and an exact model specification, we modified CNLS to minimize the sum of firms' one-sided deviations. In composite disturbance cases where noise was explicitly modeled, we extended the StoNED method to include the weak disposability axiom. The composite CNLS residuals were decomposed into noise and technical inefficiency terms and the estimated expected inefficiency was used to multiplicatively shift an average CNLS production function to obtain the weak disposability production frontier. The proposed methodology was applied to derive the technical efficiencies of 336 boilers for the U.S. coal power plants and the shadow prices of SO_2 and NO_x.

The main finding of this study is that, applying the StoNED method with a composite disturbance term, average shadow prices estimates of SO_2 are between 201 and 343 $/ton and average shadow prices of NO_x are between 409 and 1352 $/ton. Both SO_2 and NO_x shadow price estimates are in reasonable ranges comparing to allowance market prices. The proposed method can be applied to estimate shadow prices of other pollutants which can be used as references for marginal abatement costs for the industry. This marginal abatement cost is solely derived from production data so that it is not affected by market complexity.

From the results in this study, we recommend the use of weak disposability StoNED method over weak disposability DEA which is likely to overestimate shadow prices due to extreme observations. Further cost analysis tools, such as the ones proposed in this paper, will allow the EPA to investigate the outcomes of their on-going pollution control policies.

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Appendix A

Proof of Proposition 1. In a single output case, we can transform the problem (2) into an additive form:

$$\max_{\phi_o,\lambda,\mu}\left\{\varnothing_o\left|\begin{array}{ll}\sum_{i=1}^{n}\lambda_i y_i\ge y_o+\varnothing_o & \\ \sum_{i=1}^{n}\lambda_i b_{ij}\le b_{oj} & \forall j=1,\dots,J\\ \sum_{i=1}^{n}(\lambda_i+\mu_i)x_{im}\le x_{om} & \forall m=1,\dots,M\\ \sum_{i=1}^{n}(\lambda_i+\mu_i)=1 & \\ \lambda_i,\mu_i\ge 0 & \forall i=1,\dots,n\end{array}\right.\right\} \quad (19)$$

where $\theta_o=1+\frac{\varnothing_o}{y_o}$.

Applying duality theory of linear programming, the LP problem (19) has a dual problem:

$$\min_{\alpha,w,c}\left\{(\alpha+w'x_o+c'b_o)-y_o\left|\begin{array}{ll}\alpha+w'x_i+c'b_i\ge y_i & \forall i=1,\dots,n\\ \alpha+w'x_i\ge 0 & \forall i=1,\dots,n\\ w,c\ge 0 & \forall i=1,\dots,n\end{array}\right.\right\}. \quad (20)$$

We remove y_o from the objective function since it is a constant. Then we take the logarithm of the objective function and the first set of constraints, because the logarithm is a monotonic transformation for values greater than or equal to 1. Next we add the negative of $\ln y_o$ to the objective function since it is a constant. Problem (20) becomes:

$$\min_{\alpha,w,c}\left\{\ln\left(\alpha+w'x_o+c'b_o\right)-\ln y_o\left|\begin{array}{ll}\ln\left(\alpha+w'x_i+c'b_i\right)\ge \ln y_i\forall i=1,\dots,n & \\ \alpha+w'x_i\ge 0 & \forall i=1,\dots,n\\ w,c\ge 0 & \forall i=1,\dots,n\end{array}\right.\right\}. \quad (21)$$

Introducing a new variable $\epsilon_o=\ln y_o-\ln(\alpha+w'x_o+c'b_o)$ and adding an additional constraint, problem (21) can be equivalently written as:

$$\min_{\alpha,w,c,\epsilon}\left\{-\epsilon_o\left|\begin{array}{ll}\epsilon_o=\ln y_o-\ln\left(\alpha+w'x_o+c'b_o\right) & \\ \ln\left(\alpha+w'x_i+c'b_i\right)\ge \ln y_i & \forall i=1,\dots,n\\ \alpha+w'x_i\ge 0 & \forall i=1,\dots,n\\ w,c\ge 0 & \forall i=1,\dots,n\end{array}\right.\right\}. \quad (22)$$

Instead of solving (22) separately for each firm, we combine n optimization formulations and solve simultaneously for all firms. Since ϵ_i,α_i,w_i and c_i are estimated independently for each firm, we minimize the sum of ϵ_i as:

$$\min_{\alpha,w,c,\epsilon}\left\{-\sum_{i=1}^{n}\epsilon_i\left|\begin{array}{ll}\epsilon_i=\ln y_i-\ln(\alpha_i+w_i'x_i+c_i'b_i) & \forall i=1,\dots,n\\ \ln(\alpha_h+w_h'x_i+c_h'b_i)\ge \ln y_i & \forall i,h=1,\dots,n\\ \alpha_h+w_h'x_i\ge 0 & \forall i,h=1,\dots,n\\ w_i,c_i\ge 0 & \forall i=1,\dots,n\end{array}\right.\right\}. \quad (23)$$

By construction, $\epsilon_i\le 0$; we add this constraint to the problem. Moreover, we add the inefficiency term ϵ_i to the right side of the second set of constraints because of the monotonicity assumption. Note that the constraints are binding if $i=h$, and inequality otherwise:

$$\min_{\alpha,w,c,\epsilon}\left\{-\sum_{i=1}^{n}\epsilon_i\left|\begin{array}{ll}\epsilon_i=\ln y_i-\ln(\alpha_i+w_i'x_i+c_i'b_i) & \forall i=1,\dots,n\\ \ln(\alpha_h+w_h'x_i+c_h'b_i)+\epsilon_i\ge \ln y_i & \forall i,h=1,\dots,n\\ \alpha_h+w_h'x_i\ge 0 & \forall i,h=1,\dots,n\\ w_i,c_i\ge 0,\ \epsilon_i\le 0 & \forall i=1,\dots,n\end{array}\right.\right\}. \quad (24)$$

Since $\ln y_i-\epsilon_i=\ln(\alpha_i+w_i'x_i+c_i'b_i)$, the second set of constraints can be written as $\ln(\alpha_h+w_h'x_i+c_h'b_i)\ge\ln(\alpha_i+w_i'x_i+c_i'b_i)\,\forall i, h=1,\dots,n$. Removing the logarithm from this second set of constraints allows the problem (24) to be equivalently written as:

$$\min_{\alpha,w,c,\epsilon}\left\{-\sum_{i=1}^{n}\epsilon_i\left|\begin{array}{ll}\epsilon_i=\ln y_i-\ln(\alpha_i+w_i'x_i+c_i'b_i) & \forall i=1,\dots,n\\ \alpha_h+w_h'x_i+c_h'b_i\ge \alpha_i+w_i'x_i+c_i'b_i & \forall i,h=1,\dots,n\\ \alpha_h+w_h'x_i\ge 0 & \forall i,h=1,\dots,n\\ w_i,c_i\ge 0,\ \epsilon_i\le 0 & \forall i=1,\dots,n\end{array}\right.\right\} \quad (25)$$

which is the formula (6).

Proof of proposition 2. By construction, $\theta_i y_i=y_i+\varnothing_i$, thus $\theta_i=1+\varnothing_i/y_i$. By duality between the problem (19) and (20), $\varnothing_i=(\alpha_i+w_i'x_i+c_i'b_i)-y_i$. This gives $\theta_i=(\alpha_i+w_i'x_i+c_i'b_i)/y_i$. By construction the variable $\epsilon_i=\ln y_i-\ln(\alpha_i+w_i'x_i+c_i'b_i)$, thus $e^{\epsilon_i}=y_i/(\alpha_i+w_i'x_i+c_i'b_i)=1/\theta_i$.

Proof of proposition 3. Let the function $\Omega(\phi_1,\dots,\phi_n)=\sum_{i=1}^{n}(\ln y_i-\ln\phi_i)^2$. Since $\frac{\partial^2\Omega}{\partial\phi_i^2}=\frac{2}{\phi_i^2}(1-\ln\phi_i+\ln y_i)\forall i$ and $\frac{\partial^2\Omega}{\partial\phi_i\partial\phi_j}=0\forall i,j=1,\dots,n$, all non-diagonal elements in the Hessian matrix of the function Ω are equal to zero. Thus, the function Ω is convex if and only if $\frac{\partial^2\Omega}{\partial\phi_i^2}=\frac{2}{\phi_i^2}(1-\ln\phi_i+\ln y_i)\ge 0\forall i$. This condition is equivalent to $\phi_i\le e y_i\forall i$. Since the objective function of the CNLS problem (10) is a composition with an affine function $\phi_i=\alpha_i+w_i'x_i+c_i'b_i\forall i$, it is convex if the function $\Omega$is convex if and only if $\alpha_i+w_i'x_i+c_i'b_i\le e y_i\forall i$.

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