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Allocative efficiency measurement with endogenous prices

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ABSTRACT

In the nonparametric measurement of allocative efficiency, output prices are fixed. If prices are endogenous, the overall output in the market determines the allocative efficient point. We develop an alternative semi-nonparametric model that allows prices to be endogenously determined.

1. Introduction

The seminal work of Farrell (1957) introduced both the concepts of technical efficiency and allocative efficiency. Allocative efficiency assumes fixed market prices for the inputs used in the production process and for the outputs (products) generated from the production process. Much of the initial work related to production efficiency focused on the agricultural industry, where fixed prices may be a reasonable assumption due to the competitive nature of the industry and the relatively small impact any individual farmer has on prices (Farrell and Fieldhouse, 1962; Boles, 1966). However, as measuring productivity and efficiency has become common in other types of production settings, such as manufacturing and services, the same methods for estimating technical and allocative efficiencies have been used (Lovell, 1993).

A widely accepted principle in microeconomics is that firms face downward sloping demand curves; when competition is not perfect, firms' output levels influence price (Chamberlin, 1933). As researchers extend the standard nonparametric technical and allocative efficiency measurement methods to industries that are not perfectly competitive, current methods will require adjustment to consider the effects of changing output levels on prices for those outputs. This is a critical consideration, because a firm that strives to become technically efficient may actually reduce overall profits by increasing the supply of a particular product and thus reducing the market clearing price.

In this paper, we introduce an approach to model the dependency of price on output level when estimating allocative efficiency. This modeling approach is more appropriate in markets characterized by monopolistic competition than the standard nonparametric efficiency estimation models that assume perfect competition. Section 2 of this paper introduces the notation and the standard models for estimating technical and allocative efficiency. Section 3 gives an example of the different results of modeling the dependency of price on output. Finally, Section 4 concludes.

2. Nonparametric frontiers and efficiency measurement[1]

We assume that N firms produce a vector of S outputs $Y=(y_1,\dots,y_S)$ using M inputs $X=(x_1,\dots,x_M)$. We further define the observed output vector for firm i for $i=1,\dots,N$ as $Y_i=(y_{i1},\dots,y_{iS})$ and the observed input vector as $X_i=(x_{i1},\dots,x_{iM})$. Then, following Banker et al. (1984), the output-oriented technical efficiency (TE_i) of firm i is

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[1] In this paper, we focus on output expansion and revenues, holding inputs fixed. The extension to an input orientation and minimizing costs is straightforward.

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measured, assuming variable returns to scale, with the following linear program:

$$
\begin{aligned}
TE_i = & \max_{\lambda,\theta} \theta \\
s.t. & \sum_{l=1}^{N} \lambda_l y_{lj} \geq \theta y_{ij}, \qquad j = 1,\dots,S, \\
& \sum_{l=1}^{N} \lambda_l x_{lk} \leq x_{ik}, \qquad k = 1,\dots,M, \\
& \sum_{l=1}^{N} \lambda_i = 1, \\
& \lambda_l \geq 0 \quad \forall l = 1,\dots,N.
\end{aligned}
\tag{1}
$$

This measure identifies the radial distance to the frontier. Technically efficient firms may be allocatively output inefficient if the observed mix of outputs does not maximize total revenue for a given input usage. Following Färe et al. (1994), we assume that firm i faces output prices $p_i = (p_{i1},\dots,p_{iM})$.The maximum revenue R_i^* that can be obtained relative to the production technology is given by:

$$
\begin{aligned}
R_i^* = & \max_{y_j,\lambda} \sum_{j=1}^{S} p_{ij} y_j \\
s.t. & \sum_{l=1}^{N} \lambda_l y_{lj} \geq y_j, \qquad j = 1,\dots,S, \\
& \sum_{l=1}^{N} \lambda_l x_{lk} \leq x_{ik}, \qquad k = 1,\dots,M, \\
& \sum_{l=1}^{N} \lambda_l = 1, \\
& \lambda_l \geq 0 \quad \forall l = 1,\dots,N,
\end{aligned}
\tag{2}
$$

where each y_j is obtained in the solution of Eq. (2). Finally, given the observed revenue of $R_i = \sum_{j=1}^{S} p_{ij} y_{ij}$ for firm i, we can measure output revenue efficiency as $ORE_i = \frac{R_i^*}{R_i} \geq 1$. Färe et al. (1994) provide a complete decomposition of output revenue inefficiency into the technical, allocative, and scale inefficiencies.[2]

The optimal revenue obtained via Eq. (2) assumes exogenous output prices, which is consistent with perfect competition but no other market structure.[3] If in fact the firm faces a downward sloping demand curve, the results obtained in Eq. (2) will over-estimate the revenue level.

3. Example

A simple example illustrates this point. Suppose five firms $(A-E)$ are observed producing differing levels of two outputs y_1 and y_2 with the identical level of one input $x_1 = 1$. Data are presented in Table 1. As shown, only firm E is technically inefficient; relative to firm B, firm E could expand both outputs by 1.5 without increasing its input level.

We assume firm E faces demand curves $p_{E1} = 10 - 0.5y_{E1}$ and $p_{E2} = 12 - y_{E2}$ for outputs y_1 and y_2, respectively with current production, $p_{E1} = 9$ and $p_{E2} = 9.33$. Based on observed prices and output quantities, firm E's revenue is \$42.91. Solving Eq. (2) using these fixed prices, we obtain $y_1 = 3, y_2 = 4$ and $R_E^* = \$64.33$. According to this model, if E became technically efficient, its revenue would increase by \$21.42. However, this ignores the fact that increased outputs lead to lower output prices. Thus, based on the given demand curves, prices would decrease to $p_{E1} = 8.5$ and $p_{E2} = 8$.

Table 1
Illustrative Data.

Firm	y_1	y_2
A	1	5
B	3	4
C	4	3
D	5	1
E	2	2.67

If firm E became technically efficient by producing $y_1 = 3$ and $y_2 = 4$, the resulting revenue would be \$57.50, a more modest increase than suggested by Eq. (2). Of course, this occurs because of the endogenous price decreases. However, producing $y_1 = 3$ and $y_2 = 4$ is not optimal, because the output price ratio changes. In general, a firm will be able to change its output relative to Eq.(2) to accommodate the price-ratio change. We note that the solution of Eq. (2) assuming $p_{E1} = 8.5$ and $p_{E2} = 8$ is $y_1 = 4, y_2 = 3$ and $R_E^* = 58$, a reflection that the optimal mix is indeed different when prices are based on technically efficient production. Because the mix has changed with these prices, we need to solve the following non-linear program to properly identify output revenue efficiency:

$$
\begin{aligned}
R_i^* = & \max_{\lambda, y_j} \sum_{j=1}^{S} p_{ij}(y_j) y_j \\
s.t. & \sum_{l=1}^{N} \lambda_l y_{lj} \geq y_j, \qquad j = 1,\dots,S, \\
& \sum_{l=1}^{N} \lambda_l x_{lk} \leq x_{ik}, \qquad k = 1,\dots,M, \\
& \sum_{l=1}^{N} \lambda_i = 1, \\
& \lambda_l \geq 0 \quad \forall l = 1,\dots,N,
\end{aligned}
\tag{3}
$$

where $p_{ij}(y_j)$ is the demand function that depends on output y_j.[4] Returning to our example, we find the solution of Eq. (3) for firm E is:

$$p_{E1} = 8, p_{E2} = 9, \ y_1 = 4, \ y_2 = 3 \ \text{and} \ R_E^* = \$59.$$

The five-observation, two-output problem shown here is only for illustrative purposes. The data requirements discussed in the efficiency and productivity literature for nonparametric models is mixed. For example Cooper et al. (2007) argue the minimum data requirements are $n \geq \max\{m \times s, 3(m+s)\}$ based on degrees of freedom. However, Simar and Wilson (2008) argue that the asymptotic convergence rate of nonparametric estimators is much slower than those of their parametric counterparts and is influenced by the dimensionality of the estimation space. Thus, more data is necessary to estimate parameters with a similar level of confidence in nonparametric models. We recommend following the data requirements by Simar and Wilson in practical applications.

We have focused on a firm competing under a monopolistic competition where the output price is endogenous. An alternative considers the oligopoly market structure, where the output and price decisions of each firm strategically depend on the decisions of all other firms. In this case, for example, the demand function will depend on the production levels of all firms. In the long-run, changes in the scale of operation would also depend on the other firms'

[2] For our purposes, we focus on the implications of endogenous output prices without regard to the decomposition. Extending our results to this decomposition is straightforward.

[3] This is also noted by Cherchye et al. (2002).

[4] Note the model proposed in Eq. (3) is semi-nonparametric because the functional form of the demand function is assumed. However, if a nonparametric characterization of the demand function is used, the model will be fully nonparametric, see McMillan et al. (1989). Johnson and Kuosmanen (2009) discuss integrating multiple optimization problems within an efficiency setting.

impacting pricing and output decisions.[5] The oligopoly market structure strengthens our argument that using fixed prices for outputs is inappropriate, but makes the modeling more complex. The frontier benchmark which all firms would try to achieve (in the classical Farrell framework) would be technically and allocatively efficient (in the production sense), leading to a cost-minimizing mix of inputs and production at the most productive scale.[6]

4. Conclusions

This paper extends the models available for measuring technical and allocative efficiencies to settings of imperfect competition where output prices are endogenously determined. Thus, a firm attempts to maximize its own revenue and the resulting output levels are consistent with profit maximization, yet are not socially optimal. In our monopolistic competition example, firms use market power to restrict their output levels, leading to a deadweight loss. The new model described captures the relationship between price and output level when measuring allocative efficiency.

Current efficiency studies tend to focus on a static estimation of cross-sectional efficiency. However, analysts often use these results to give advice regarding technical and allocative efficiency strategies. This paper implies that moving towards this static allocatively efficient benchmark when prices are endogenous is not appropriate. Rather, changes in output prices should be taken into account when identifying an allocative and technically efficient benchmark. We suggest that future research could consider a game theoretic model with both interdependent and endogenous prices.

References

Banker, R.D., Charnes, A., Cooper, W.W., 1984. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science 30 (9), 1078–1092.

Boles, J.N., 1966. Efficiency Squared-Efficient Computation of Efficiency Indexes. Western Farm Economic Association, Los Angles, Pullman. Annual Meeting.

Chamberlin, E., 1933. The Theory of Monopolistic Competition. Harvard University Press, Cambridge.

Cherchye, L., Kuosmanen, T., Post, G.T., 2002. Non-parametric production analysis in non-competitive environments. International Journal of Production Economics 80 (3), 279–294.

Cooper, W.W., Seiford, L.M., Tone, K., 2007. Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software. Springer, New York, NY, USA.

Färe, R., Grosskopf, S., Lovell, C.A.K., 1994. England. Cambridge University Press, New York, NY, USA.

Farrell, M.J., 1957. The measurement of productive efficiency. Journal of the Royal Statistical Society. Series A. General 120, 253–281.

Farrell, M.J., Fieldhouse, M., 1962. Estimating efficient productions functions under increasing returns to scale. Journal of the Royal Statistical Society. Series A. General 125, 252–267.

Johnson, A.L., Kuosmanen, T., 2009. How Operational Conditions and Practices Affect Productive Performance? Efficient Semi-Parametric One-Stage Estimators. http://ssrn.com/abstract=1485733 2009 Available at SSRN.

Lovell, C.A.K., 1993. Production frontiers and productive efficiency. In: Fried, H.O., Lovell, C.A.K., Schmidt, S.S. (Eds.), The Measurement of Productivity and Efficiency. Oxford University Press, New York.

McMillan, J., Ullah, A., Vinod, H.D., 1989. Estimation of the shape of the demand curve by nonparametric kernel methods. Advances in Econometrics and Modeling. Dordrech, Kluwer, pp. 85–92.

Simar, L., Wilson, P.W., 2008. Statistical inference in nonparametric frontier models: Recent developments and perspectives. In: Fried, H.O., Lovell, C.A.K., Schmidt, S.S. (Eds.), The Measurement of Productive Efficiency and Productivity Growth. Oxford University Press, New York, pp. 421–451.

[5] We thank an anonymous referee for making this point.

[6] This would likely be the Cournot n-firm Nash equilibrium solution. However, a significant extension of Cournot competition to multiple outputs would be necessary. We leave this extension for future research.