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A note on picker blocking models in a parallel-aisle order picking system

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This note develops analytical picker blocking models to simply and accurately assess picker blocking in parallel-aisle order picking systems when multiple picks occur at a pick point. The Markov chain–based models characterize the two bounding walking speeds for modeling picker movement: unit walk time and instantaneous walk time. The unit walk time model has a state-space transition matrix that is reduced by a factor of 16 for both narrow-aisle and wide-aisle systems. Additionally, the model improves upon the existing literature by providing a closed-form expression for the narrow-aisle system with instantaneous walk time. Experimental results are provided to demonstrate how picker blocking is influenced by pick density in a variety of scenarios under varying assumptions regarding the maximum number of picks at a pick point. These results broaden those previously presented in the literature, as well as demonstrate the improved efficiency of the proposed model.[1]

Keywords: Warehousing, picker blocking, discrete-time Markov chains

1. Introduction

Parallel-aisle bin shelving Order Picking Systems (OPSs) are one of the most popular alternatives for order picking in distribution centers due to minimal investment costs while providing reconfigurability (Frazelle, 2002). Pickers circumnavigate aisles to retrieve items from shelves and place them on carts (or vehicles). A typical system is illustrated in Fig. 1. Using this simple picker-to-part mechanism, warehouse managers have considerable operational flexibility including staffing more pickers when demand is expected to increase. Alternatively, multiple orders can be aggregated into batches to improve picking efficiency by reducing the number of trips needed to retrieve the set of orders. Both adding pickers and batching orders are expected to enhance throughput. However, *picker blocking* can occur when multiple pickers traverse the same pick area, and this congestion reduces the benefits derived from adding pickers (Ruben and Jacobs, 1999).

Picker blocking occurs in two ways: (i) when multiple pickers traverse a pick area while maintaining a no passing restriction (Gue *et al.*, 2006; Parikh and Meller, 2010) and (ii) two or more pickers attempt to occupy the same space or the same resource simultaneously (Parikh and Meller, 2009). When a picker prevents another picker from passing, *in-the-aisle blocking* arises as depicted in Fig. 2(a); when pickers attempt to pick from the same storage location, *pick-point blocking* occurs as depicted in Fig. 2(b).

Here we follow the terminology established in Parikh and Meller (2010) for narrow-aisle picking systems. A picking area is modeled as a set of pick points. A pick point is a position in the aisle where the picker stops to pick; because of the narrow-aisle characteristic multiple pick locations are blocked on both sides of the aisle. Thus, at every pick point, a picker has access to multiple Stock Keeping Units (SKUs) and blocks access to those same SKUs for other pickers.

In narrow-aisle OPSs, pickers follow one-way routes through the aisles, whereas in a wide-aisle OPS, two-way routes can be used. In-the-aisle blocking is associated with narrow-aisle OPS and pick-point blocking occurs with both narrow- and wide-aisle systems.

[1]A shorter and less technical version of this article was published previously as "Analysis of Picker Blocking in Narrow-Aisle Batch Picking" in *Progress in Material Handling Research: Proceedings of 2010 International Material Handling Research Colloquium*.
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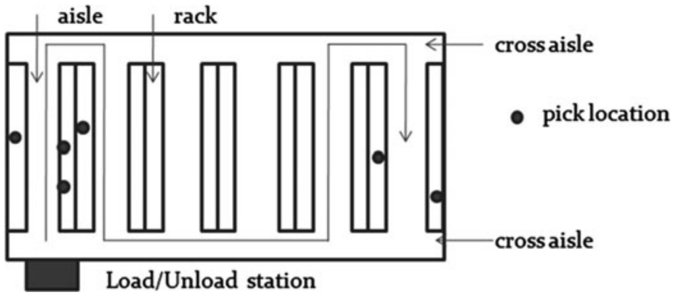

Fig. 1. A typical parallel-aisle OPS and a routing example (modified from Gademann and Van de Velde (2005)).

Several researchers (Skufca, 2005; Gue *et al.*, 2006; Parikh and Meller, 2009, 2010) have investigated the effects of picker blocking and developed analytical models to estimate the time lost due to blocking. However, the current literature on blocking models when multiple picks can occur at a pick point—i.e., the wide-aisle models of Parikh and Meller (2009) and the narrow-aisle models of Parikh and Meller (2010)—exhibits two modeling drawbacks: complex Markov chain models and lack of closed-form expressions. In a unit walk time case—i.e., an extremely slow walk speed—the analytical model has added complexity due to the explicit tracking of the operational state of each picker. In addition, for the instantaneous walk-time model in a narrow-aisle OPS, no closed-form solution was previously presented. Closed-form expressions for both extreme walk time situations are valuable to identify the range of throughput rates and analyze the convergence characteristics of the system.

This note develops and uses analytical models where multiple picks can be made at a pick point. Previously, Gue *et al.* (2006) found that a batch picking strategy in narrow-aisle OPSs is valuable when the pick time is constant at a pick point. Parikh and Meller (2010) found that picker blocking can be significant when the variation in pick density is high; e.g., when the pick time at a pick point varies perhaps due to multiple picks at a pick point. Similarly, wide-aisle OPSs may also experience significant blocking when each pick point requires multiple picks (Parikh and Meller, 2009).

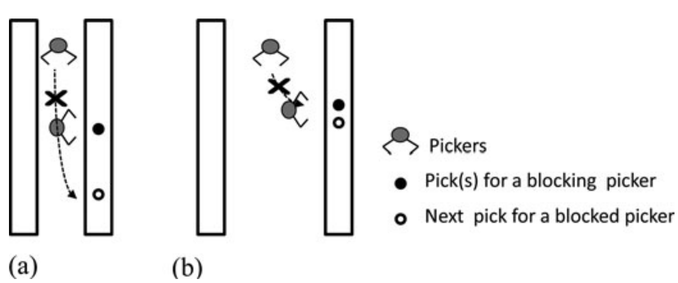

Fig. 2. Types of picker blocking: (a) in-the-aisle (picker) blocking and (b) pick-point blocking (Parikh and Meller, 2009).

This note improves analytical models of picker blocking in two ways: (i) significant reduction in complexity of the Markov chain model for unit walk time; and (ii) closed-form expression for instantaneous walk models in narrow-aisle OPSs. The experimental results also provide new insight into how picker blocking is influenced by pick density across a range of scenarios for both narrow- and wide-aisle systems and under varying assumptions regarding the maximum number of picks at a location. The remainder of this note is organized as follows. We define the problem domain, including the types of picker blocking, in Section 2. Section 3 presents a new analytical model and defines closed-form expressions for solving the narrow-aisle model. A simulation comparison in Section 4 produces several insights into practical order picking environments. Section 5 extends the analytical model to a wide-aisle OPS assuming a pick : walk time ratio of unity. We summarize the findings and offer suggestions for future research in Section 6.

2. Problem definition

2.1. *Picker blocking models using a circular-aisle abstraction*

We use a circular-aisle abstraction shown in Fig. 3. This abstraction was introduced by Skufca (2005) and applied by Gue *et al.* (2006) to model a narrow-aisle OPS and adapted by Parikh and Meller (2009) to model a wide-aisle OPS. We consider two order pickers and make the following assumptions.

1. The circular order picking aisle consists of n pick points.
2. The order pickers take a one-way traversal route, meaning that they travel through that aisle in only one direction (in the circular model this implies that they move only in a clockwise direction).
3. Pick time is constant regardless of the pick point characteristics such as shelf height.
4. At a pick point, pickers pick with a probability p; q denotes $1 - p$, the probability of walking past a pick

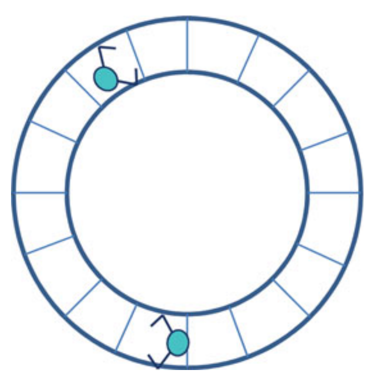

Fig. 3. A circular order picking aisle (Gue *et al.*, 2006) (color figure provided online).

point; a picker picks again at the same pick point with probability p independent of the number of previous picks (i.e., if p is large, the number of picks at a pick point is potentially large; however, this will be relaxed in the simulation study in Section 4).
5. The potential states of a picker are picking, walking, or standing idle due to blocking.
6. The pick time, pt, and the walk time between two pick points, wt, are deterministic.

Note that since a pick point contains multiple SKUs the probability of picking is not the probability of picking any one of the SKUs but rather one minus the probably of not picking any of the SKUs stored at that pick point (assume each SKU is only stored in one location).

The pick time here refers to the picking of a single SKU, which may include multiple items. The assumption of a constant deterministic pick time is more accurate when the number of items picked for any SKU is relatively low (and thus all items can be retrieved by accessing the storage location once), the SKUs are similar in size and shape, and the storage locations require similar levels of effort to access. A constant deterministic pick time is a common assumption in the literature; however, this will be relaxed in the simulation study in Section 4.

An important parameter of analytical models for picker blocking is the ratio of the time to pick at a point to the time to walk past that point, which is referred to as the pick : walk time ratio. The current literature focus on two extreme pick : walk time ratios: the unit walk time case (pick : walk time = 1:1) and the instantaneous walk time case (pick : walk time = 1:0)). These two cases should bound all practical situations, which, in general, are between 1:0.05 to 1:0.2 (Gue *et al.*, 2006).

As a performance measure, we use the percentage of time blocked, denoted as $b^{\mathrm{n}}_{pt:wt}(k)$ and $b^{\mathrm{w}}_{pt:wt}(k)$, where n and w stand for a narrow aisle (i.e., both in-the-aisle blocking and pick-point blocking can occur) and a wide aisle (i.e., only pick-point blocking can occur), respectively. k is the number of pickers in the system, where for these models $k = 2$.

3. Picker blocking models in a narrow-aisle system

In this section, we develop new Markov chain models that improve the narrow-aisle models of Parikh and Meller (2010) for both the pick : walk time 1:1 and 1:0 cases. Picker blocking refers to in-the-aisle blocking unless otherwise stated.

3.1. *Pick : walk time = 1:1*

Let D_t denote the distance between picker 1 and picker 2 at time t. Assume the pick : walk time ratio is 1:1, the distance $d \in D_t$ can be expressed as

$$(n + [(\text{picker 1 position}) - (\text{picker 2 position})]) \bmod n, \quad (1)$$

and ranges from unity to $n - 1$. A Markov chain is introduced by defining state $S_t = 0$ (*block*) representing picker 1 blocking picker 2, state $S_t = n$ representing picker 2 blocking picker 1; states $[1, 2, \ldots, n - 1]$ are given by $S_t = D_t$. All states can be summarized by the vector [blocked, $1, 2, \ldots, n - 1$, blocked]. These states allow us to distinguish four transition cases:

1. *Transition probabilities between unblocked states.*

If both pickers pick ($p \times p$) or walk ($q \times q$), the current distance (D_t) does not change at $t + 1$. However, when picker 1 picks while picker 2 walks ($p \times q$), the distance decreases by one. When picker 1 walks while picker 2 picks ($q \times p$), the distance increases by one.

2. *Transition probabilities from an unblocked state to a blocked state.*

When the distance from picker 1 to picker 2 is unity, a blocked state can arise if picker 1 picks (with probability p), and picker 2 walks (with probability q). *Vice versa*, when the distance from picker 1 to picker 2 is $n - 1$, the current state becomes a blocked state if picker 1 walks (with probability q) and picker 2 picks (with probability p).

3. *Transition probabilities from a blocked state to an unblocked state.*

If picker 1 is blocked by picker 2, picker 1 must wait for picker 2 to walk (with probability q) to exit a blocked state and *vice versa*.

4. *Transition probabilities between blocked states.*

When the current state is blocked, a pick can occur with probability p. Blocking status remains; i.e., a blocked state transits to a blocked state with probability p.

The transition diagram characterizing the state-space and transition probabilities is shown in Fig. 4. Parikh and Meller (2010) conditioned on the picker's current operation (walking or picking) as well as the current distance to obtain the Markov property. Including the operation-related states of two pickers requires four more states per distance; i.e., picker 1's pick and walk and picker 2's pick and walk states. Our proposed discrete-time Markov chain of picker blocking for multiple picks with a pick : walk time = 1:1 is not conditioned on the pickers' current operation (walking or picking). A Markov property of distance holds regardless of the previous walking or picking status and for both the single- and multiple-pick cases.

We define a transition matrix (**A**) having states [*block*, 1, $2, \ldots, n - 2, n - 1$, *block*] by [*block*, $1, 2, \ldots, n - 2, n - 1$, *block*] and a vector (**v**) having states [*block*, $1, 2, \ldots, n - 2$, $n - 1$, *block*]. Parikh and Meller's (2010) transition matrix has dimensions $16 \times (n - 1) \times (n - 1)$; however, the model

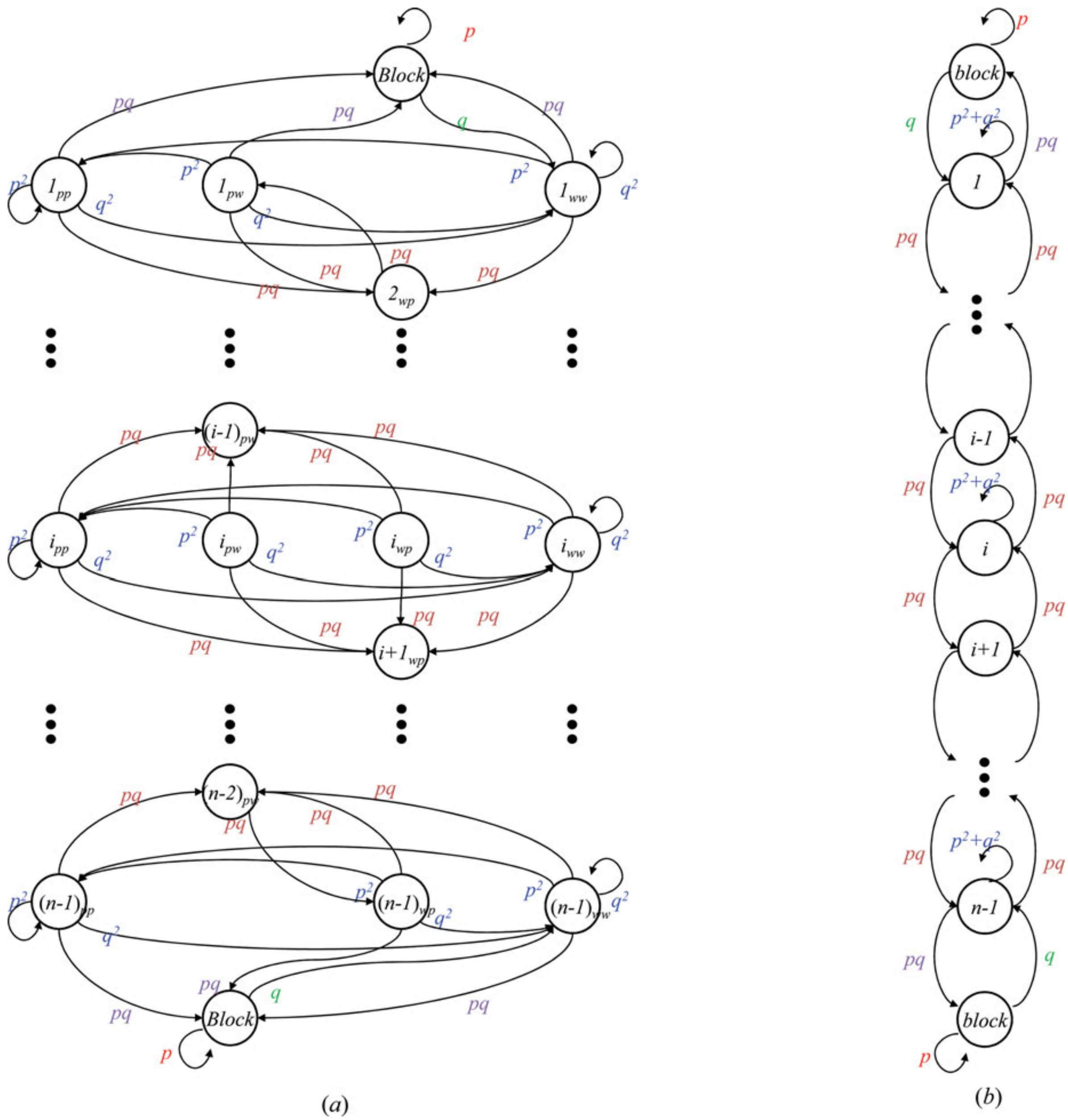

Fig. 4. State space and transitions for the Markov chain model when the picking time equals travel time: (a) the model of Parikh and Meller (2010) and (b) our model (color figure provided online).

we present has dimensions $(n+1)\times(n+1)$, which is a significant reduction:

$$\mathbf{A}=\begin{bmatrix} p & q & 0 & \cdots & 0 & 0 & 0\\ pq & p^2+q^2 & pq & \cdots & 0 & 0 & 0\\ 0 & pq & p^2+q^2 & \ddots & \ddots & 0 & 0\\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots\\ 0 & 0 & \ddots & \ddots & p^2+q^2 & pq & 0\\ 0 & 0 & 0 & \cdots & pq & p^2+q^2 & pq\\ 0 & 0 & 0 & \cdots & 0 & q & p \end{bmatrix}.$$

Stationary distribution

We obtain:

$$\begin{aligned}\mathbf{v} &= [blocked, 1, 2, \ldots, n-1, blocked]\\ &= [1, 1/p, 1/p, \ldots, 1/p, 1]\end{aligned}$$

that satisfies $\mathbf{vA}=\mathbf{v}$. The stationary density using $\|\mathbf{v}\|$ is scaled to obtain a stationary probability as $2\times 1+(n-1)/p = 2+(n-1)/p$. The blocking probability of one picker in a blocked state is

$$b^{\mathrm{n}}_{1:1}(2)=\frac{p}{2p+n-1}, \tag{2}$$

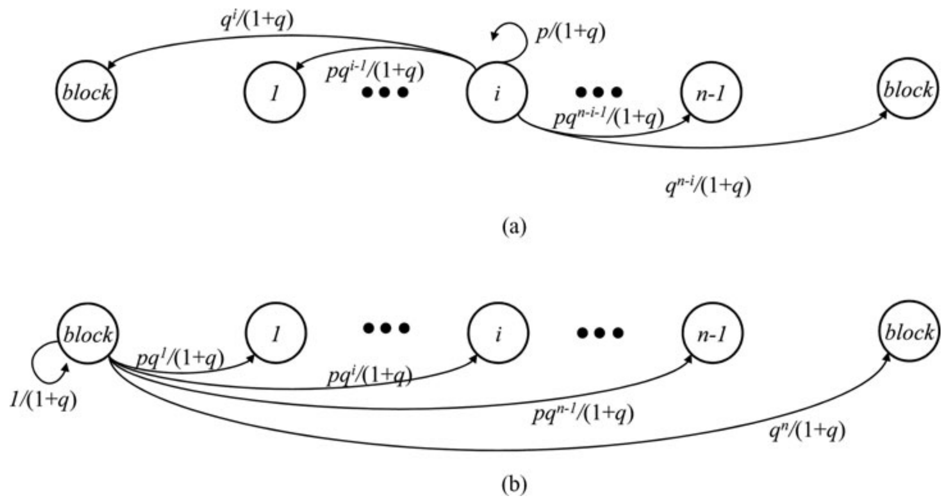

Fig. 5. State spaces and transitions for the Markov chain model when the travel time is infinite: (a) unblocked case and (b) blocked case.

which is equivalent to that presented in Parikh and Meller (2010).

3.2. *Pick : walk time = 1:0*

The infinite speed assumptions allow for transitions to multiple states in our Markov chain model. Thus, the probability for each picker moving a distance x is approximated and used to develop an estimate of a probability function for the distance y that characterizes the change in the distance between the two pickers.

Let random variables X_t^1 and X_t^2 represent the desired number of locations moved in time t by pickers 1 and 2, respectively, and $Y_t = X_t^1 - X_t^2$ denote the change in distance between the two pickers when passing is not allowed. As described in Parikh and Meller (2009) and Hong *et al.* (2010), the probability density function of Y_t (i.e., $g(y)$) is

$$g(y) = \frac{pq^{|y|}}{1+q} \quad \text{for} -\infty < y < \infty. \tag{3}$$

Suppose the distance at the previous state is $D_{t-1} = r$. The actual change in distance is bounded by the physical blocking phenomenon and the value r. Two transition cases are defined.

1. *Transition probabilities to unblocked states.*

In this case, the distribution function (3) is used directly. Note that r is zero or n when a picker is blocked. Thus, the change with given r is within one to $n-1$:

$$P(Y_t = y) = g(y) = \frac{pq^{|y|}}{1+q}$$
$$\text{for } 1-r < y < n-1-r, r = 0, \ldots, n.$$

2. *Transition probabilities to blocked states.*

The next step is to calculate the probability of events with blocking. To obtain this probability, we need to accumulate all cases beyond the limits (0 or n). We note that there will be blocking at state 0 if $Y_t \leq -r$. $g(y)$ is symmetric and can be calculated as

$$P(Y_t \geq r) = \sum_{y=r}^{\infty} \frac{pq^{|y|}}{1+q} = \frac{p}{1+q} q^r \frac{1}{1-q} = \frac{q^r}{1+q},$$
$$P(Y_t \geq n-r) = \frac{q^{n-r}}{1+q}.$$

Figure 5 illustrates two examples. Conditional on the current state, alternative transition models are used. While managing the transition from *blocked* (0) to *blocked* (n) or *blocked* (n) to *blocked* (0), we use $q^n/(1+q)$ for the transition probability.

The result forms the transition matrix:

$$\mathbf{A} = \begin{bmatrix} \frac{1}{1+q} & \frac{pq}{1+q} & \frac{pq^2}{1+q} & \cdots & \frac{pq^{n-2}}{1+q} & \frac{pq^{n-1}}{1+q} & \frac{q^n}{1+q} \\ \frac{q}{1+q} & \frac{p}{1+q} & \frac{pq}{1+q} & \cdots & \frac{pq^{n-3}}{1+q} & \frac{pq^{n-2}}{1+q} & \frac{q^{n-1}}{1+q} \\ \frac{q^2}{1+q} & \frac{pq}{1+q} & \ddots & \ddots & \ddots & \frac{pq^{n-3}}{1+q} & \frac{q^{n-2}}{1+q} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \frac{q^{n-2}}{1+q} & \frac{pq^{n-3}}{1+\beta} & \ddots & \ddots & \ddots & \frac{pq}{1+q} & \frac{q^2}{1+q} \\ \frac{q^{n-1}}{1+q} & \frac{pq^{n-2}}{1+\beta} & \frac{pq^{n-3}}{1+\beta} & \cdots & \frac{pq}{1+q} & \frac{p}{1+q} & \frac{q}{1+q} \\ \frac{q^n}{1+q} & \frac{pq^{n-1}}{1+q} & \frac{pq^{n-2}}{1+q} & \cdots & \frac{pq^2}{1+q} & \frac{pq}{1+q} & \frac{1}{1+q} \end{bmatrix}.$$

Stationary distribution

To determine a stationary distribution, a $\mathbf{v}$ that satisfies $\mathbf{vA} = \mathbf{v}$ is identified as

$\mathbf{v} = [blocked, 1, 2, \ldots, n-1, blocked] = [1, p, p, \ldots, p, 1].$

We can scale the stationary density using $\|\mathbf{v}\| = 2 + (n-1)p$. Thus, the blocking probability of a picker in one of the blocked states is

$$b^{\mathrm{n}}_{1:0}(2) = \frac{1}{2 + (n-1)\,p}. \tag{4}$$

This closed-form expression does not appear in the previous literature to the best of our knowledge. The new state space of pick : walk time = 1:0 contributes to deriving the closed-form expression of Equation (4). We validate the analytical model in Equation (4) via a comparison with simulation models (see Appendix A for details).

3.3. *Convergence characteristics*

From the closed-form expressions and simulation models, we observe the convergence characteristics over increasing pick densities (see Appendix B for a summary of the results) and make the following observation.

Theorem 1. *In a two-picker OPS, the percentage of time blocked converges to* $1/(n+1)$ *as the pick density approaches unity.*

Proof. This proof is a direct extension of the previously presented results. When the walk speed is equal to the pick time, we can take the limit of Equation (2) as the pick density approaches one as

$$\lim_{p\to 1} \frac{p}{n + 2p - 1} = \frac{1}{n+1}.$$

When pickers walk at infinite speed, Equation (4) converges to the same value:

$$\lim_{p\to 1} \frac{1}{2 + (n-1)\,p} = \frac{1}{n+1}.$$

This completes the proof. ∎

4. A simulation study

In practice, the pick time is stochastic and the maximal number of picks at a pick point is typically not large. Because there are multiple SKUs stored at a pick point and multiple orders within a batch may request the same SKUs, a picker may need to make multiple picks at the same location. The pick times can be stochastic due to variability in pickers' abilities and characteristics of the pick location. Further shelf space will limit the maximum number of picks at a pick point. This section presents the results of a simulation study that analyzes the impacts of stochastic pick times and assuming the number of picks at a pick point is limited.

Table 1. The average gap (%) and maximum gap (%) between the analytical models and stochastic pick time models

Pick time distribution	*wt = 1*		*wt = 0*	
	Avg.	*Max*	*Avg.*	*Max*
Uni	0.39	1.19	0.49	1.09
Tri	2.25	5.04	1.95	3.83
Exp	4.21	9.06	3.29	5.87

4.1. *The stochastic pick time*

We consider three pick time distributions: Uni = uniform [min, max] = [0.5, 1.5], Tri = triangular [min, mode, max] = [0.5, 1.5, 1.0], and Exp = exponential [mean] = [1.0], where the time unit represents a time spent to retrieve a SKU. The results are compared and summarized in Table 1. We conclude from Table 1 that stochastic pick times have little effect on the analytical results when the upper bound on the number of SKUs picked at a pick point is unbounded. The largest deviations in total pick time are less than 10% and typically closer to 3 or 4%.

4.2. *The finite number of picks*

In Section 3, we assumed that the maximum number of picks at a pick point is unbounded. However, in practice, the number of picks at a pick point is finite because of the capacity of the shelves. Let m denote the number of picks at a pick point. According to Parikh and Meller (2009), the expected number of picks during a tour is $I = n \times [(p - m \times p^m + (m-1) \times p^{(m+1)})/(1-p) + m \times p^m]$, where n is the number of possible pick points in the picking area. Table 2 shows the impacts of M, an *a priori* defined maximal number of picks at a pick point, on the relationship between p, the probability of picking (typically assumed to be uniform over the entire picking area), and I, the expected number

Table 2. The expected number of picks (I) in a trip when a picker is made at a pick point with probability = p and the maximal number of SKUs at a pick point = M in a 20-pick point circular OPS

	M				
p	*10*	*20*	*50*	*100*	*Infinite*
0.01	0.2	0.2	0.2	0.2	0.2
0.10	2.2	2.2	2.2	2.2	2.2
0.20	5.0	5.0	5.0	5.0	5.0
0.30	8.6	8.6	8.6	8.6	8.6
0.40	13.3	13.3	13.3	13.3	13.3
0.50	20.0	20.0	20.0	20.0	20.0
0.60	29.8	30.0	30.0	30.0	30.0
0.70	45.3	46.6	46.7	46.7	46.7
0.80	71.4	79.1	80.0	80.0	80.0
0.90	117.2	158.1	179.1	180.0	180.0
0.99	189.3	360.5	782.1	1255.3	1980.0

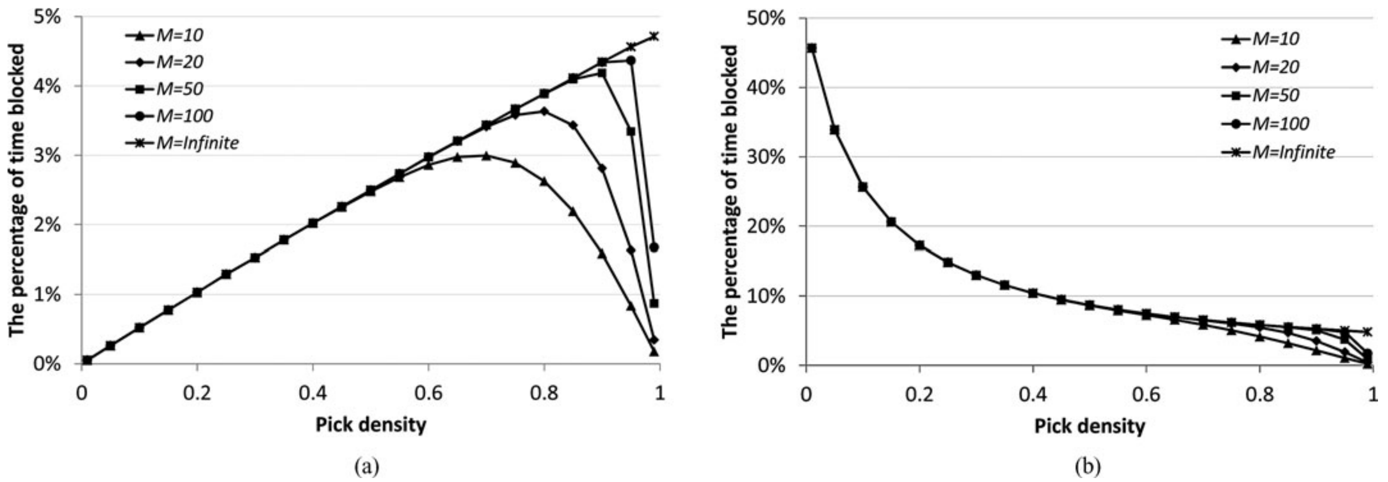

Fig. 6. The percentage of time blocked for both pick : walk time = 1 and pick : walk time = 0 varying the number of picks at a pick point. Results for a 20-pick points-two picker system: (a) pick : walk time = 1 and (b) pick : walk time = 0.

of picks in a trip. When p is less than 0.5, the impacts are not significant. However, as p becomes greater than 0.5, I is affected by the assumption regarding the maximal number of picks at a pick point. A high probability of picking is more common in smaller picking areas and areas with fast moving products.

When the number of SKUs picked at a pick point is limited, Theorem 1 no longer holds. Figure 6 shows that as the pick density increases, the percentage of time blocked converges to zero, not $1/(n+1)$. Note that no picker blocking arises when the pick density equals exactly zero and the percentage of time blocked equals to $1/(n+1)$ only when there is no limit on the number of picks at a point.

Observation 1: When the number of SKUs picked at a pick point is finite (i.e., M is finite) in a two-picker OPS, the percentage of time blocked converges to zero as the pick density approaches unity.

4.3. *Comparison simulation*

We considered a triangular distribution for pick times and assumed that the maximum number of picks at a pick point was 20. This simulation study considered pick-to–walk time ratios of 1:0.05, 1:0.1, and 1:0.2; the number of pick points as 10, 20, and 50; and the number of workers = 2, 5, and 10. Results are summarized in Fig. 7 for pick-to–walk time variation, in Fig. 8 for the number of pickers, and in Fig. 9 for the number of pick points.

We note that under these assumptions initially high levels of picker blocking exist, but as the pick density increases beyond 0.75, the percentage of time blocked falls and approaches zero. When the pick density is low, the variability in pick times increases picker blocking compared with a scenario with deterministic pick times. However, as the pick density increases beyond 0.75, the non-deterministic situation experiences less blocking delay because limiting the number of picks at a pick point creates less variation. As pick density approaches unity, the non-deterministic situation shows a smaller loss in productivity from picker blocking.

Figure 7 also shows that as pickers travel more quickly, picker blocking becomes a bigger managerial issue. In particular, the impact of walk speed is more acute and higher with relatively low pick densities (i.e., $p = [0, 0.5]$). As expected, Fig. 8 shows that picker blocking increases as the number of pickers increases. These increases are less pronounced for the wide-aisle system as we will see in Section 5. Though increased staffing may be considered as a means to increase order picking throughput, the pickers' utilization can drop significantly because of picker blocking and must be considered in the throughput analysis. Figure 9 indicates that the number of pick points in a system can also

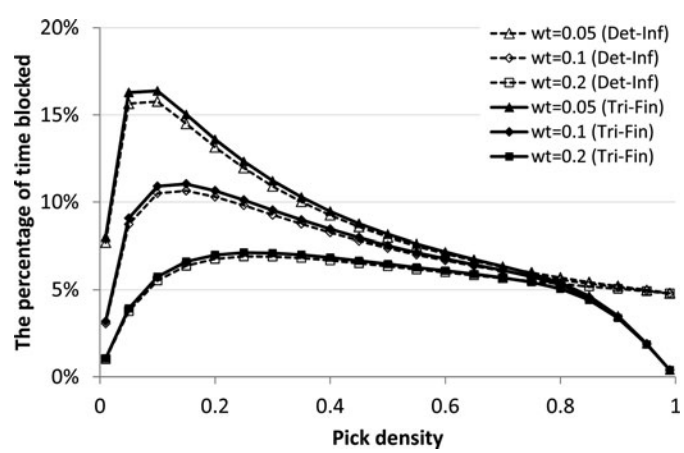

Fig. 7. The percentage of time blocked considering various walk times for both deterministic pick time and infinite M (Det-Inf) and pick times follow a triangular distribution and $M = 20$ (Tri-Fin). Results for a 20-pick points-two-picker system.

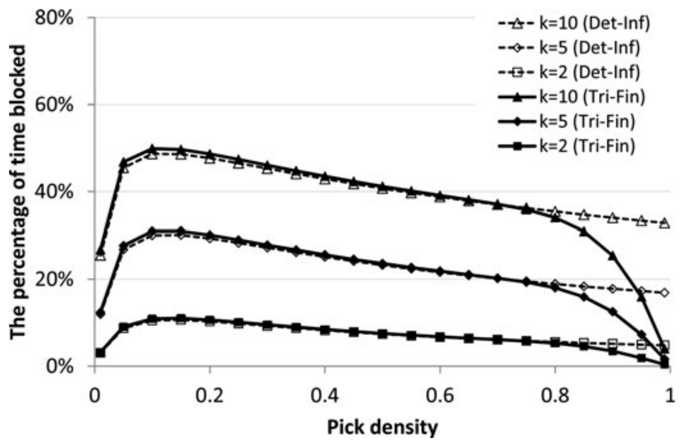

Fig. 8. The percentage of time blocked considering varying the number of pickers for both deterministic pick time and infinite M (Det-Inf) and pick time = Tri and $M = 20$ (Tri-Fin). Results assuming a pick : walk time ratio = 1:0.1 for 20 pick points.

affect picker blocking, since it impacts the spread among pickers.

5. An application: picker blocking model when pick : walk time = 1:1 in a wide-aisle system

The wide-aisle study of Parikh and Meller (2009) demonstrated a Markov property when transitioning from the picker's current operation (walking or picking) and the current distance. Thus, we can again apply the succinct model described in Section 3 to demonstrate the applicability of the reduced state space model to alternative OPSs.

Similar to the narrow-aisle model, D_t denotes the distance between picker 1 and picker 2 at time t. Assume the pick : walk time ratio is 1:1, the distance $d \in D_t$ can be expressed by Equation (1) and ranges from zero to $n-1$. A Markov chain defines state $S_t = block$ representing picker 1 blocking picker 2 or picker 2 blocking picker 1; states $[0, 1, \ldots, n-1]$ are given by $S_t = D_t$. All states can be summarized by the vector $[block, 0, 1, 2, \ldots, n-1]$. These states allow us to distinguish four transition cases. Again, when multiple picks are allowed, a Markov property of distance holds regardless of the previous walking or picking status.

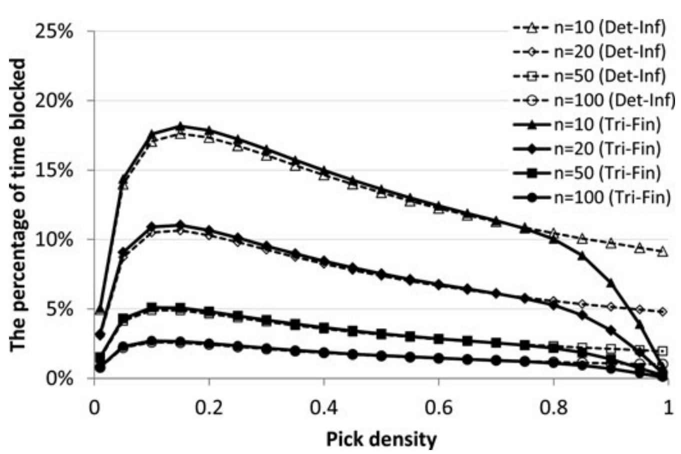

Fig. 9. The percentage of time blocked considering varying the number of pick points for both deterministic pick time and infinite M (Det-Inf) and pick time = Tri and $M = 20$ (Tri-Fin). Results assuming a pick : walk time ratio = 1:0.1 for a two-picker system.

1. *Transition probabilities between unblocked states.*

If both pickers pick ($p \times p$) or walk ($q \times q$), the current distance (D_t) does not change at $t+1$. However, when picker 1 picks while picker 2 walks ($p \times q$), the distance decreases by one. When picker 1 walks while picker 2 picks ($q \times p$), the distance increases by one.

2. *Transition probabilities from an unblocked state to a blocked state.*

When the distance from picker 1 to picker 2 is zero, a blocked state can arise if both pickers pick (with probability $p \times p$). We assume that the choice of blocking and blocked pickers is randomly determined. *Vice versa*, if both pickers walk ($q \times q$), the current distance (D_t) does not change at $t+1$.

3. *Transition probabilities from a blocked state to an unblocked state.*

A blocked state can be caused by either picker 1 blocking picker 2 or picker 2 blocking picker 1. With probability 0.5 picker 1 is blocked. If picker 1 is blocked by picker 2, picker 1 must wait for picker 2 to walk (with probability q) to exit a blocked state; thus, the probability is $q/2$.

4. *Transition probabilities between blocked states.*

When the current state is blocked, a pick can occur with probability p. Blocking status remains; i.e., a blocked state transits to a blocked state with probability p.

Figure 10 illustrates the state space and their transitions.

We define a transition matrix ($\mathbf{A}$) having states $[block, 0, 1, 2, \ldots, n-2, n-1]$ by $[block, 0, 1, 2, \ldots, n-2, n-1]$ and a vector ($\mathbf{v}$) having states $[block, 0, 1, 2, \ldots, n-2, n-1]$. The transition matrix of Parikh and Meller (2009) has dimensions $16 \times n \times n$; however, the model we present has dimensions $(n+1) \times (n+1)$:

$$\mathbf{A} = \begin{bmatrix} p & \frac{q}{2} & 0 & \cdots & 0 & 0 & \frac{q}{2} \\ p^2 & q^2 & pq & \cdots & 0 & 0 & pq \\ 0 & pq & p^2+q^2 & \ddots & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \ddots & \ddots & p^2+q^2 & pq & 0 \\ 0 & 0 & 0 & \cdots & pq & p^2+q^2 & pq \\ 0 & pq & 0 & \cdots & 0 & qq & p^2+q^2 \end{bmatrix}.$$

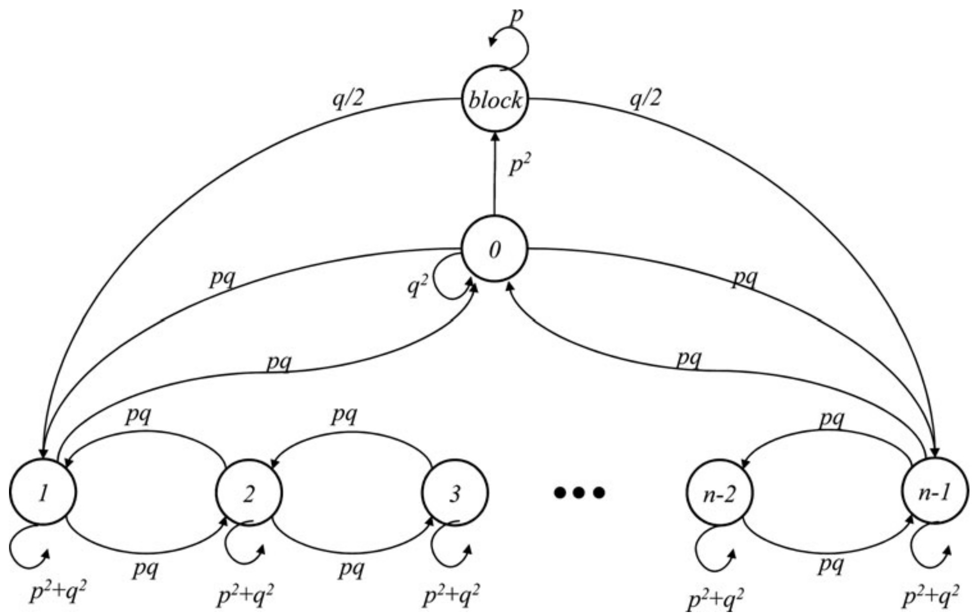

Fig. 10. State space and transitions for the Markov chain model when picking time equals travel time in a wide-aisle situation with multiple-pick allowance.

Stationary distribution

We obtain the **v** satisfying $\mathbf{vA} = \mathbf{v}$:

$$\mathbf{v} = [\mathit{blocked}, 0, 1, \ldots, n-1] = \left[1, \frac{q}{p^2}, \frac{1+q}{2p^2}, \ldots, \frac{1+q}{2p^2}\right].$$

We can scale the stationary density using $\|\mathbf{v}\|$ to obtain a stationary probability. From **v** above, we have

$$\|\mathbf{v}\| = 1 + \frac{q}{p^2} + (n-1)\,\frac{1+q}{2p^2} = \frac{2p^2 + 2q + (n-1)(1+q)}{2p^2} = \frac{2p^2 - p + n\,(2-p)}{2p^2}.$$

The blocking probability of blocking state of a picker is

$$b^{\mathrm{w}}_{1:1}(2) = \frac{v_{1*}}{\|\mathbf{v}\|} = \frac{\frac{1}{2}}{(2p^2 - p + n(2-p))/2p^2} = \frac{p^2}{2p^2 - p + n(2-p)}. \quad (5)$$

Equation (5) is identical to the result in Parikh and Meller (2009) but results from a much smaller transition matrix. Thus, the succinct model provides an improved foundation to build models for other scenarios such as instances of pick : walk time ratios of 1:0.5 and 1:0.25.

6. Conclusion and further study

In this note, an analytical study on picker blocking in narrow-aisle systems was introduced. This study identifies simpler, more efficient closed-form analytical models for a multiple-pick order picking model. The simpler model was also applied in a wide-aisle system. With wide aisles, picker blocking is reduced but can still be significant under certain scenarios and system characteristics. The use of a Markov chain model can be applied in a wide variety of scenarios; e.g., situations with more than two pickers and non-extreme walk speed cases. Our model provides a more suitable foundation for expanding to consider these broader set of cases.

References

Frazelle, E. (2002) *World-Class Warehousing and Material Handling*, McGraw-Hill, New York, NY.

Gademann, N. and Van de Velde, S. (2005) Order batching to minimize total travel time in a parallel-aisle warehouse. *IIE Transactions*, **37**(1), 63–75.

Gue, K.R., Meller, R.D. and Skufca, J.D. (2006) The effects of pick density on order picking areas with narrow aisles. *IIE Transactions*, **38**(10), 859–868.

Hong, S., Johnson, A.L. and Peters, B.A. (2010) Analysis of picker blocking in narrow-aisle batch picking, in *Progress in Material Handling Research: Proceedings of 2010 International Material Handling Research Colloquium*, The Material Handling Institute, Charlotte, NC, pp. 366–378.

Parikh, P.J. and Meller, R.D. (2009) Estimating picker blocking in wide-aisle order picking systems. *IIE Transactions*, **41**, 232–246.

Parikh, P.J. and Meller, R.D. (2010) A note on worker blocking in narrow-aisle order picking systems when pick time is non-deterministic. *IIE Transactions*, **42**(6), 392–404.

Ruben, R.A. and Jacobs, F.R. (1999) Batch construction heuristics and storage assignment strategies for walk/ride and pick systems. *Management Science*, **45**(4), 575–596.

Skufca, J.D. (2005) *k*-Workers in a circular warehouse: a random walk on a circle, without passing. *SIAM Review*, **47**(2), 301–314.

Appendices

Appendix A: Comparison of analytical and simulation models

Table A1 summarizes the simulation results validating the analytical models. The results of the 1:1 analytical model are identical to the model by Parikh and Meller (2010). The results for the 1:0 analytical models differ from their results by 0.032–0.170%. The gap between the performances of the simulation model and the analytical model is 0.01–0.33% for the metric of time blocked (Diff% = (the percentage of time blocked by the analytical model – the percentage of time blocked by the simulation model)/(the percentage of the time blocked by the analytical model) × 100). There is one exception: when picker blocking occurs rarely, for example when $p = 0.05$ in pick : walk time = 1:1, the difference between the simulation results and the analytical model are slightly more pronounced. These results show that the analytical model reasonably approximates a multiple-pick OPS.

Appendix B: The percentage of time blocked for different pick : walk time ratios

We conducted a simulation study with pick : walk time = 1:0.025, 1:0.05, 1:0.1, 1:0.2, and 1:0.5. Figure A1(a) illustrates the simulations' results of a two-picker model and Fig. A1(b) a five-picker model. The solid lines are the results with pick : walk time = 1:0, 1:0.025, 1:0.05, 1:0.1, 1:0.2, 1:0.5, and 1:1 from top to bottom. The upper dotted line is an analytical result with pick : walk time = 1:0. The lower dotted line is an analytical result with pick : walk time = 1:1.

As pick density increases, the percentage of time blocked converges to approximately the value derived in Theorem 1, $1/(n+1)$. For example, when $p = 0.95$ shown in Fig. A1(a),

Table A1. Comparison of analytical and simulation results of the percentage of time blocked in a circular aisle (20 pick points)

Probability	*Pick : walk time = 1:1*			*Pick : walk time = 1:0*		
p	*Analytical*	*Simulation*	*Diff %*	*Analytical*	*Simulation*	*Diff %*
0.05	0.2618	0.2580	1.43	33.8983	33.8823	0.05
0.1	0.5208	0.5225	−0.33	25.6410	25.6283	0.05
0.2	1.0309	1.0313	−0.03	17.2414	17.2454	−0.02
0.3	1.5306	1.5256	0.33	12.9870	12.9916	−0.04
0.4	2.0202	2.0186	0.08	10.4167	10.4181	−0.01
0.5	2.5000	2.5005	−0.02	8.6957	8.6871	0.10
0.6	2.9703	2.9655	0.16	7.4627	7.4567	0.08
0.7	3.4314	3.4243	0.21	6.5359	6.5327	0.05
0.8	3.8835	3.8749	0.22	5.8140	5.8007	0.23
0.9	4.3269	4.3154	0.27	5.2356	5.2224	0.25
0.95	4.5455	4.5491	−0.08	4.9875	4.9917	−0.08

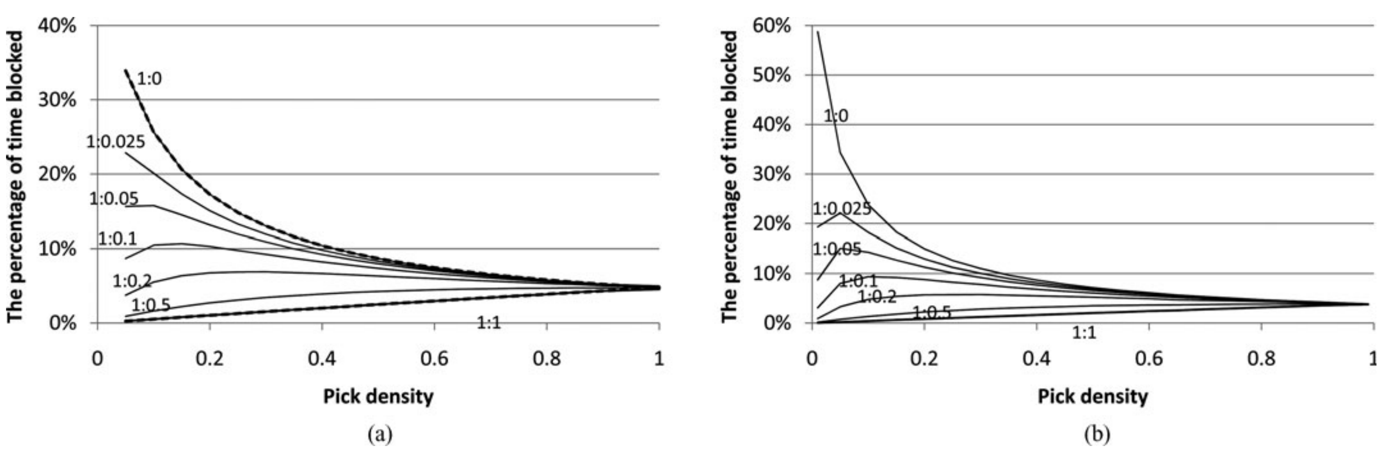

Fig. A1. The percentage of time blocked over different pick : walk time ratios: (a) two pickers in 20 pick points and (b) five pickers in 100 pick points.

the percentage of throughput loss due to picker blocking ranges between 4.53 and 5.00 in a 20-pick point circular picking system with two pickers. Theorem 1 approximates the loss as $1/21 = 4.76$. Figure A1(b) converges to [3.79, 3.86].

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